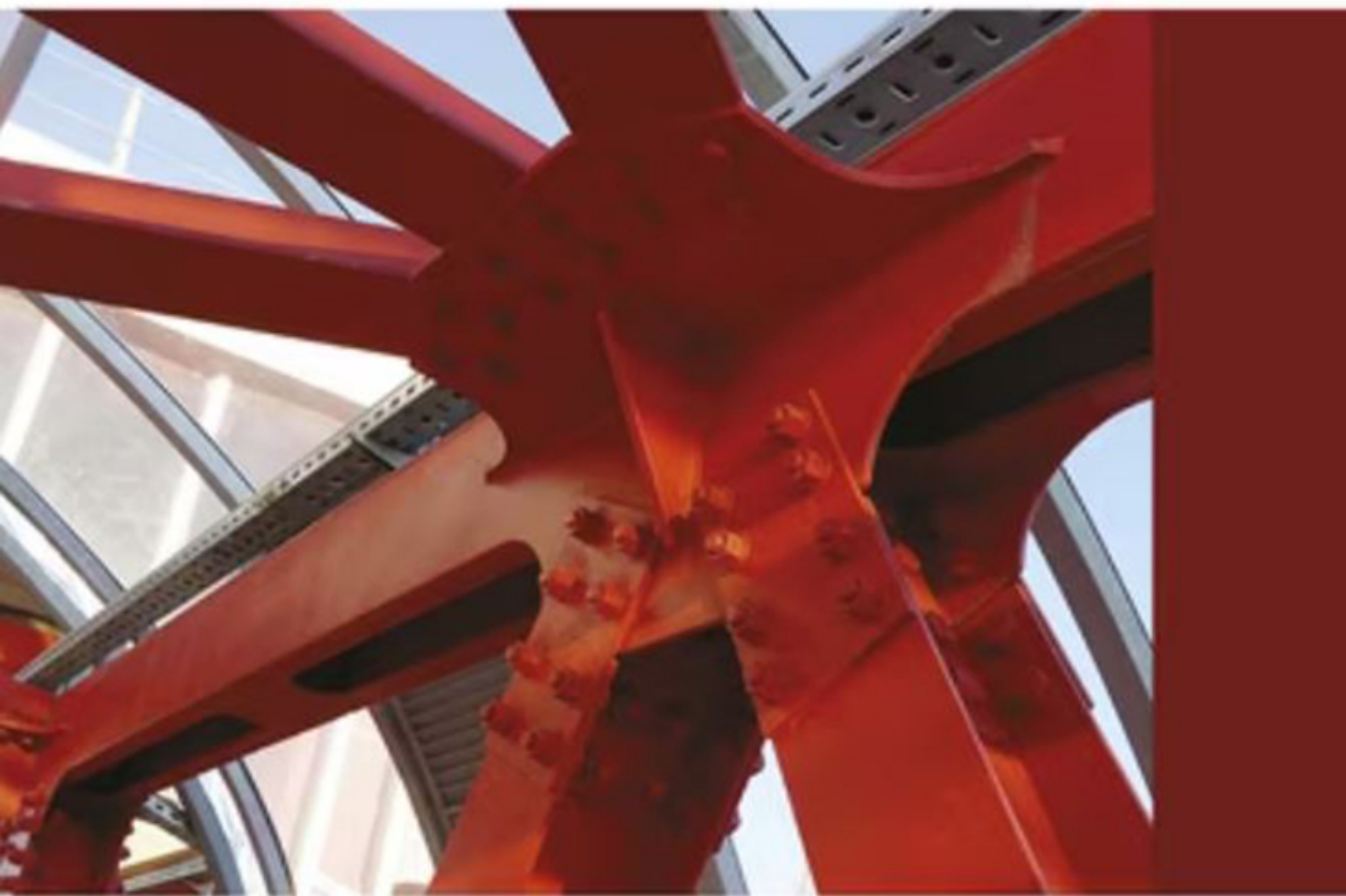


# MECHANICS OF MATERIALS

Eleventh Edition



R. C. HIBBELER

# MECHANICS OF MATERIALS

ELEVENTH EDITION

R. C. HIBBELER



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It is intended that this book provide the student with a clear and thorough presentation of the theory and application of the principles of mechanics of materials. To achieve this objective, over the years this work has been shaped by the comments and suggestions of hundreds of reviewers in the teaching profession, as well as many of the author's students. The eleventh edition has been significantly enhanced from the previous edition, and it is hoped that both the instructor and student will benefit greatly from these improvements.

---

## New to this Edition

- **Expanded Answer Section.** The answer section in the back of the book now includes additional information related to the solution of select Fundamental Problem in order to offer the student some guidance in solving the problems.
- **Re-writing of Text Material.** Further clarification of some concepts has been included in this edition, and throughout the book the accuracy has been enhanced, and important definitions are now in boldface throughout the book to highlight their importance.
- **New Photos.** The relevance of knowing the subject matter is reflected by the real-world applications depicted in the over 14 new or updated photos placed throughout the book. These photos generally are used to explain how the relevant principles apply to real-world situations and how materials behave under load.
- **New Problems.** There are approximately 30% new problems that have been added to this edition, which involve applications to many different fields of engineering.
- **New Videos.** Three types of videos are available that are designed to enhance the most important material in the book. Lecture videos serve to test the student's ability to understand the concepts. Example problem videos are intended to review these problems, and fundamental problem videos guide the student in solving these problems that are in the book.

---

## Contents

The subject matter is organized into 14 chapters. Chapter 1 begins with a review of the important concepts of statics, followed by a formal definition of both normal and shear stress, and a discussion of normal stress in axially loaded members and average shear stress caused by direct shear.

In Chapter 2 normal and shear strain are defined, and in Chapter 3 a discussion of some of the important mechanical properties of materials is given. Separate treatments of axial load, torsion, and bending are presented in Chapters 4, 5, and 6, respectively. In each of these chapters, both linear-elastic and plastic behavior of the material covered in the previous chapters, where the state of stress results from combined loadings. In Chapter 9 the concepts for transforming multiaxial states of stress are presented. In a similar manner, Chapter 10 discusses the methods for strain transformation, including the application of various theories of failure. Chapter 11 provides a means for a further summary and review of previous material by covering design applications of beams and shafts. In Chapter 12 various methods for computing deflections of beams and shafts are covered. Also included is a discussion for finding the reactions on these members if they are statically indeterminate. Chapter 13 provides a discussion of column buckling, and lastly, in Chapter 14 the problem of impact and the application of various energy methods for computing deflections are considered.

Sections of the book that contain more advanced material are indicated by a star (\*). Time permitting, some of these topics may be included in the course. Furthermore, this material provides a suitable reference for basic principles when it is covered in other courses, and it can be used as a basis for assigning special projects.

**Alternative Method of Coverage.** Some instructors prefer to cover stress and strain transformations *first*, before discussing specific applications of axial load, torsion, bending, and shear. One possible method for doing this would be first to cover stress and its transformation, Chapter 1 and Chapter 9, followed by strain and its transformation, Chapter 2 and the first part of Chapter 10. The discussion and example problems in these later chapters have been styled so that this is possible. Also, the problem sets have been subdivided so that this material can be covered without prior knowledge of the intervening chapters. Chapters 3 through 8 can then be covered with no loss in continuity.

---

## Hallmark Elements

**Organization and Approach.** The contents of each chapter are organized into well-defined sections that contain an explanation of specific topics, illustrative example problems, and a set of homework problems. The topics within each section are placed into subgroups defined by titles. The purpose of this is to present a structured method for introducing each new definition or concept and to make the book convenient for later reference and review.

**Chapter Contents.** Each chapter begins with a full-page illustration that indicates a broad-range application of the material within the chapter. The “Chapter Objectives” are then provided to give a general overview of the material that will be covered.

**Procedures for Analysis.** Found after many of the sections of the book, this unique feature provides the student with a logical and orderly method to follow when applying the theory. The example problems are solved using this outlined method in order to clarify its numerical application. It is to be understood, however, that once the relevant principles have been mastered and enough confidence and judgment have been obtained, the student can then develop his or her own procedures for solving problems.

**Photographs.** Many photographs are used throughout the book to enhance conceptual understanding and explain how the principles of mechanics of materials apply to real-world situations.

**Important Points.** This feature provides a review or summary of the most important concepts in a section and highlights the most significant points that should be realized when applying the theory to solve problems.

**Example Problems.** All the example problems are presented in a concise manner and in a style that is easy to understand.

**Homework Problems.** Apart from the preliminary, fundamental, and conceptual problems, there are numerous standard problems in the book that depict realistic situations encountered in engineering practice. It is hoped that this realism will both stimulate the student’s interest in the subject and provide a means for developing the skill to reduce any such problem from its physical description to a model or a symbolic

representation to which principles may be applied. Throughout the book there is an approximate balance of problems using either SI or FPS units. Furthermore, in any set, an attempt has been made to arrange the problems in order of increasing difficulty. The answers to all but every fourth problem are listed in the back of the book. To alert the user to a problem without a reported answer, an asterisk (\*) is placed before the problem number. Answers are reported to three significant figures, even though the data for material properties may be known with less accuracy. Although this might appear to be a poor practice, it is done simply to be consistent, and to allow the student a better chance to validate his or her solution.

**Appendices.** The appendices of the book provide a source for review and a listing of tabular data. Appendix A provides information on the centroid and the moment of inertia of an area. Appendices B and C list tabular data for structural shapes, and the deflection and slopes of various types of beams and shafts.

**Accuracy.** As with the previous editions, apart from the author, the accuracy of the text and problem solutions has been thoroughly checked in part by Kai Beng Yap, a practicing engineer, and a team of specialists at EPAM, including Georgii Kolobov, Ekaterina Radchenko, and Artur Akberov.

---

## Acknowledgments

Over the years, this book has been shaped by the suggestions and comments of many of my colleagues in the teaching profession. Their encouragement and willingness to provide constructive criticism are very much appreciated and it is hoped that they will accept this anonymous recognition. There are a few people, however, that I feel deserve particular mention. They are S. Ahmad, A. Asgharatal, K. Dennehy, A. Lutz, M. Walter, and M. Zhang. A special note of thanks also goes to Jun Hwa Lee, who provided a careful reading of the manuscript, and checked many of the problems. Through the years, however, Kai Beng Yap supported me in this regard, but unfortunately his support has come to an end, due to his untimely passing. His contribution to this effort, and his friendship will be deeply missed.

During the production process I am also thankful for the assistance of Rose Kernan, my production editor for many years, and to my wife, Conny, for her help in proofreading of the manuscript during production.

Finally, I would also like to thank all my students who have used the previous edition and have made comments to improve its contents.

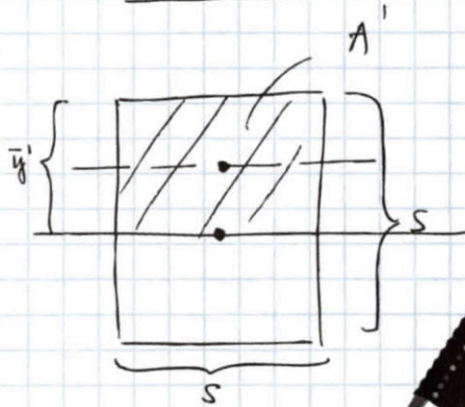
I would greatly appreciate hearing from you if at any time you have any comments or suggestions regarding the contents of this edition.

Russell Charles Hibbeler  
*hibbeler@bellsouth.net*



# your work...

PART A



$$S = 6.75 \text{ in}$$

$$\bar{y}' = \frac{S}{2} = \frac{6.75 \text{ in}}{2} = 3.375 \text{ in}$$

$$A' = S \times 0.5 \times S = 6.75 \text{ in} \times 0.5 \times 6.75 \text{ in} = 22.8 \text{ in}^2$$

$$Q = \bar{y}' A' = 3.375 \text{ in} \times 22.8 \text{ in}^2 = 76.9 \text{ in}^3$$

$$Q = 76.9 \text{ in}^3$$

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□ μA ↶ ↷ ↻ ⌨ ?

Q =

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[Hints](#) [My Answers](#) [Give Up](#) [Review Part](#)

**Incorrect; Try Again; 5 attempts remaining**

The distance between the horizontal centroidal axis of area  $A'$  and the neutral axis of the beam's cross section is half the distance between the top of the shaft and the neutral axis.

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## Resources for Instructors

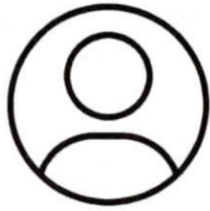
**Instructor's Solutions Manual** This supplement provides complete solutions supported by problem statements and problem figures. The Instructor's Solutions Manual is available in PDF format on Pearson Higher Education website: [www.pearson.com](http://www.pearson.com).

**PowerPoint Slides** A complete set of all the figures and tables from the textbook are available in PowerPoint format.

---

## Resources for Students

**Videos** Developed by the author, three different types of videos are now available to reinforce learning the basic theory and applying the principles. The first set provides a lecture review and a self-test of the material related to the theory and concepts presented in the book. The second set provides a self-test of the example problems and the basic procedures used for their solution. And the third set provides an engagement for solving the Fundamental Problems throughout the book. For more information on how to access these videos visit [www.pearson.com/hibbeler](http://www.pearson.com/hibbeler).



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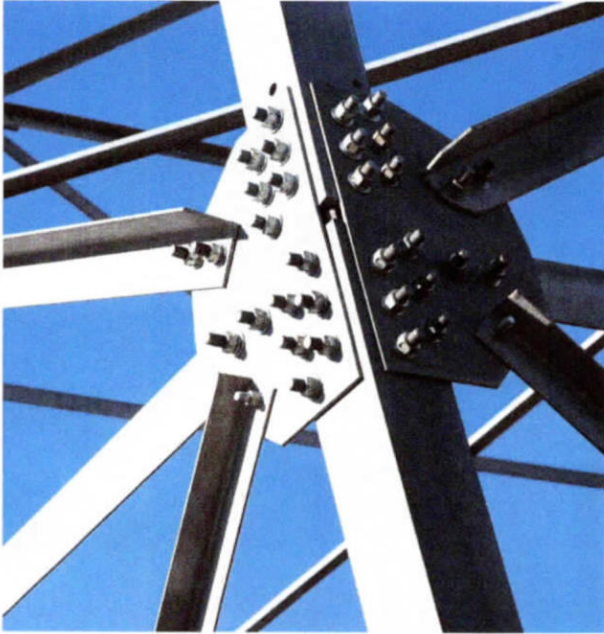
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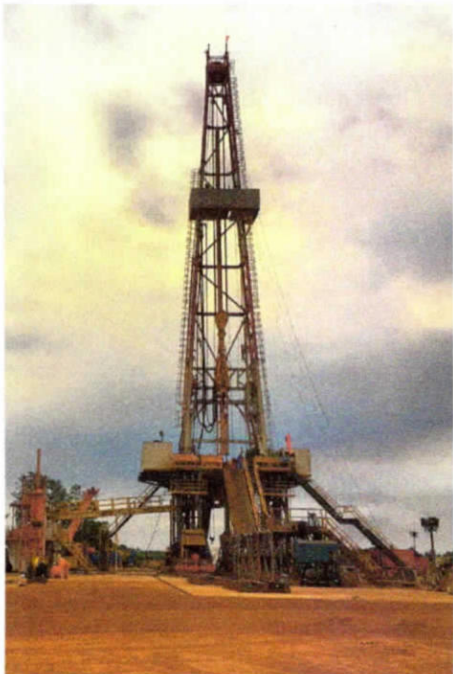
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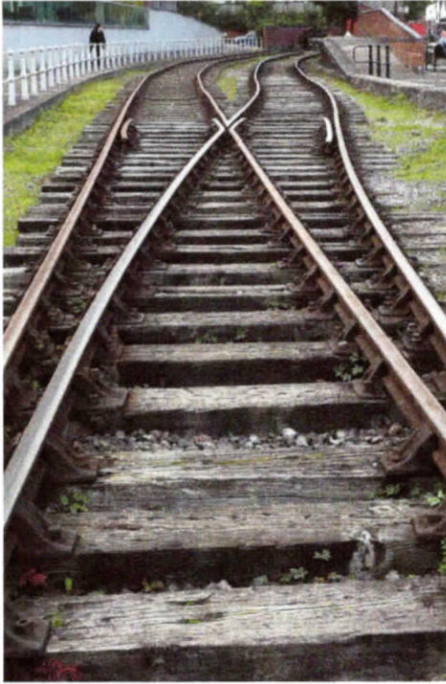
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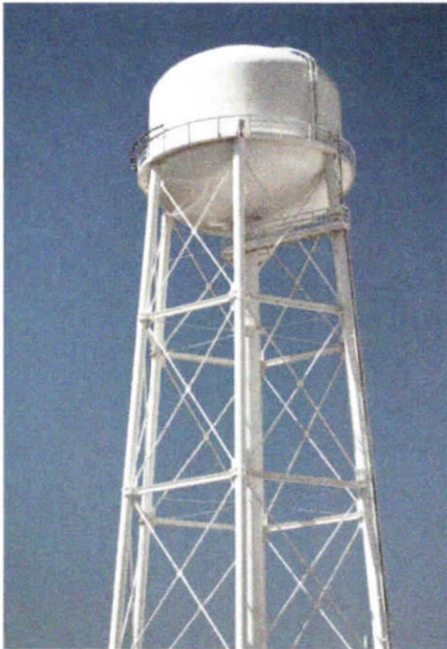
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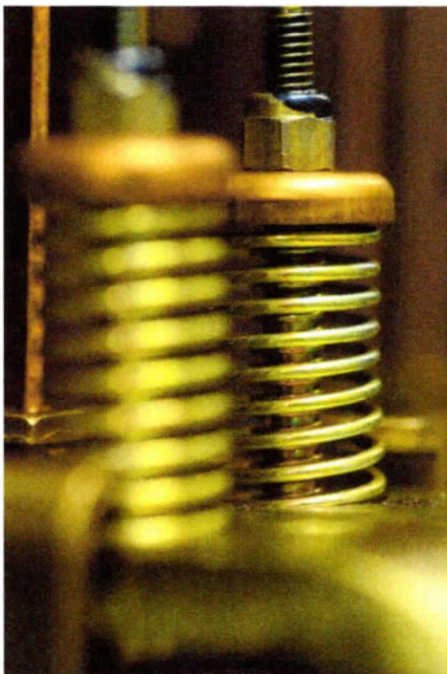
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# MECHANICS OF MATERIALS

ELEVENTH EDITION

# CHAPTER 7



The bolts used for the connections of this steel framework are subjected to stress. In this chapter we will discuss how engineers design these connections and their fasteners.

# STRESS

## CHAPTER OBJECTIVES

- In this chapter we will review some of the important principles of statics and show how they are used to determine the internal loadings in a body. Afterwards the concepts of normal and shear stress will be introduced, and applications of the analysis and design of members subjected to an axial load or direct shear will be discussed.

## 1.1 INTRODUCTION

*Mechanics of materials* is a branch of mechanics that studies the internal effects of stress and strain in a solid body. Stress is the result of internal loading and so it is related to the strength of the material, while strain is a measure of the deformation produced by the internal loadings. A thorough understanding of the fundamentals of this subject is of vital importance for the design of any machine or structure, because many of the formulas and rules of design cited in engineering codes are based upon the principles of this subject.



**Historical Development.** The origin of mechanics of materials dates back to the beginning of the seventeenth century, when Galileo Galilei performed experiments to study the effects of loads on rods and beams made of various materials. However, it was not until the beginning of the nineteenth century when experimental methods for testing materials were vastly improved. At that time many experimental and theoretical studies in this subject were undertaken, primarily in France, by such notables as Saint-Venant, Poisson, Lamé, and Navier.

Through the years, after many fundamental problems had been solved, it became necessary to use advanced mathematical and computer techniques to solve more complex problems. As a result, mechanics of materials has expanded into other areas of mechanics, such as the *theory of elasticity* and the *theory of plasticity*.

## 1.2 EQUILIBRIUM OF A DEFORMABLE BODY

Since statics plays an important role in both the development and application of mechanics of materials, it is very important to have a good understanding of its fundamentals. For this reason we will now review some of the main principles of statics that will be used throughout the book.

**Loads.** A body can be subjected to both surface loads and body forces. **Surface loads** that act on a small area of contact are reported by **concentrated forces**, while **distributed loadings** act over a larger surface area of the body. When the loading is coplanar, as in Fig. 1-1a, then a resultant force  $F_R$  of a distributed loading is equal to the area under the distributed loading diagram, and this resultant acts through the geometric center or centroid of this area.

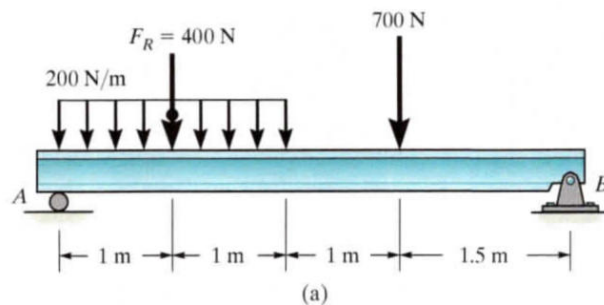
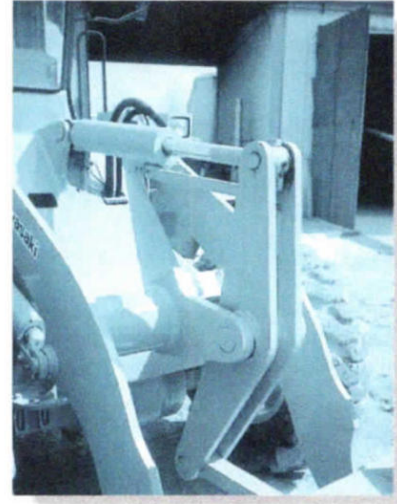


Fig. 1-1

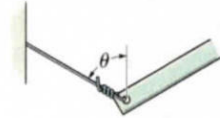
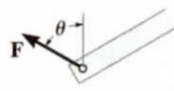
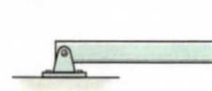
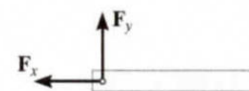


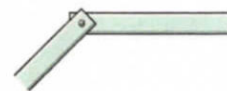
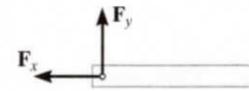
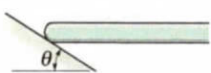


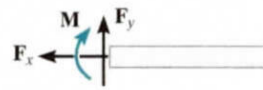
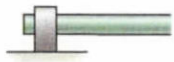


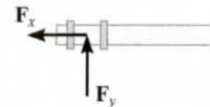
A **body force** is developed when one body exerts a force on another body without direct physical contact between the bodies. Examples include the effects caused by the earth's gravitation or by an electromagnetic field. Although these forces affect all the particles composing the body, they are normally represented by a single concentrated force acting on the body. In the case of gravitation, this force is called the **weight  $W$**  of the body and acts through the body's center of gravity.

**Support Reactions.** For bodies subjected to coplanar force systems, the supports most commonly encountered are shown in Table 1–1. Whatever the support, as a general rule, **if the support prevents translation in a given direction, then a force must be developed on the member in that direction. Likewise, if rotation is prevented, a couple moment must be exerted on the member.** For example, the roller support only prevents translation perpendicular or normal to the surface. Hence, the roller exerts a normal force  $F$  on the member at its point of contact. Since the member can freely rotate about the roller, a couple moment cannot be developed on the member.



Many machine elements are pin connected in order to enable free rotation at their connections. These supports exert a force on a member, but no moment.

TABLE 1–1

Type of connection	Reaction	Type of connection	Reaction
 <p>Cable</p>	 <p>One unknown: <math>F</math></p>	 <p>External pin</p>	 <p>Two unknowns: <math>F_x, F_y</math></p>
 <p>Roller</p>	 <p>One unknown: <math>F</math></p>	 <p>Internal pin</p>	 <p>Two unknowns: <math>F_x, F_y</math></p>
 <p>Smooth support</p>	 <p>One unknown: <math>F</math></p>	 <p>Fixed support</p>	 <p>Three unknowns: <math>F_x, F_y, M</math></p>
 <p>Journal bearing</p>	 <p>One unknown: <math>F</math></p>	 <p>Thrust bearing</p>	 <p>Two unknowns: <math>F_x, F_y</math></p>



In order to design the members of this building frame, it is first necessary to find the internal loadings at various points along their length.

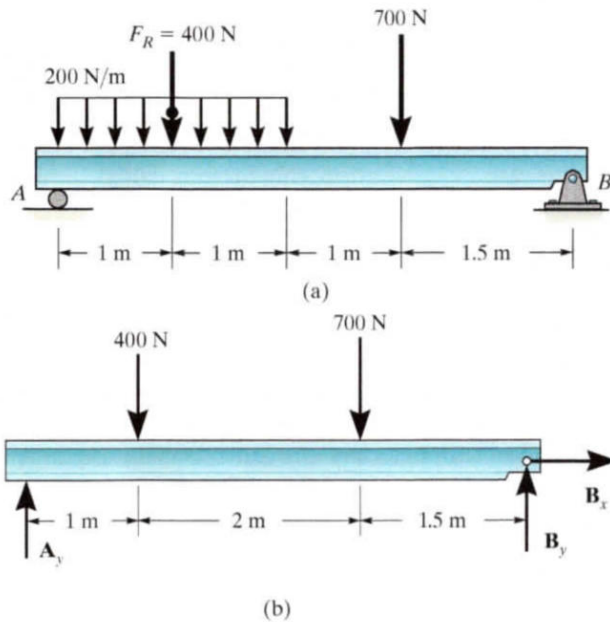


Fig. 1-1 (cont.)

**Equations of Equilibrium.** Equilibrium of a body requires both a *balance of forces*, to prevent the body from translating or having accelerated motion along a straight or curved path, and a *balance of moments*, to prevent the body from rotating. These conditions are expressed mathematically as the equations of equilibrium:

$$\begin{aligned} \Sigma \mathbf{F} &= \mathbf{0} \\ \Sigma \mathbf{M}_O &= \mathbf{0} \end{aligned} \quad (1-1)$$

Here,  $\Sigma \mathbf{F}$  represents the sum of all the forces acting on the body, and  $\Sigma \mathbf{M}_O$  is the sum of the moments of all the forces about any point  $O$  either on or off the body.

If an  $x, y, z$  coordinate system is established with the origin at point  $O$ , the force and moment vectors can be resolved into components along each coordinate axis, and the above two equations can be written in scalar form as six scalar equations, namely,

$$\begin{aligned} \Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0 \\ \Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0 \end{aligned} \quad (1-2)$$

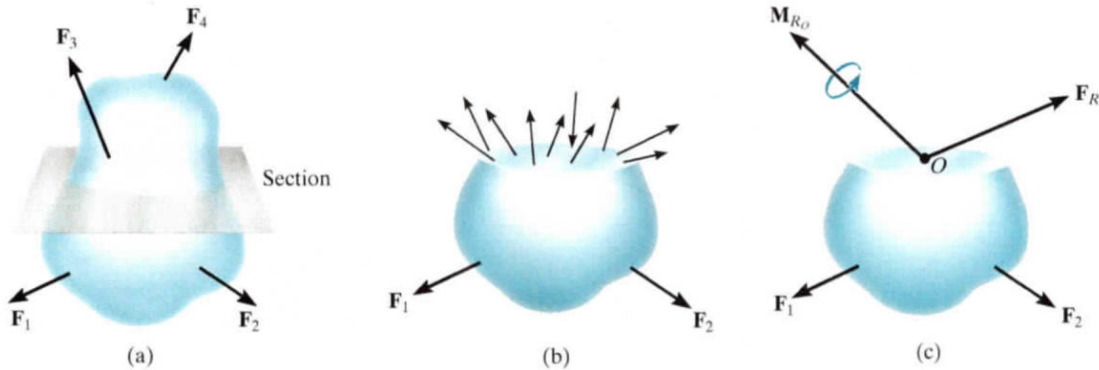


Fig. 1-2

Often in engineering practice the loading on a body can be represented as a system of *coplanar forces* in the  $x$ - $y$  plane. In this case equilibrium of the body can be specified with only three scalar equilibrium equations, that is,

$$\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma M_O &= 0 \end{aligned} \quad (1-3)$$

Successful application of the equations of equilibrium must include all the known and unknown loadings that act on the body, and the best way to account for these loadings is to draw the body's free-body diagram first. For example, the free-body diagram of the beam in Fig. 1-1*a* is shown in Fig. 1-1*b*. Here each force is identified by its magnitude and direction. Included are the body's dimensions in order to sum the moments of the forces.

**Internal Resultant Loadings.** In mechanics of materials, statics is primarily used to determine the internal loadings that act within a body. This is done using the *method of sections*. For example, consider the body shown in Fig. 1-2*a*, which is held in equilibrium by the four external forces.\* In order to obtain the internal loadings acting on a specific region within the body, it is necessary to pass an imaginary section through the region where these loadings are to be determined. The two parts of the body are then separated, and a free-body diagram of one of the parts is drawn. If we consider the bottom segment, there will be a distribution of internal force acting on the "exposed" area of the section, Fig. 1-2*b*. These forces actually represent the effects of the material of the top section of the body acting on the bottom section.

Although the exact distribution of this internal loading may be *unknown*, its resultants  $\mathbf{F}_R$  and  $\mathbf{M}_{R,O}$  can be determined by applying the equations of equilibrium to the segment shown in Fig. 1-2*c*. Here these loadings act at point  $O$ ; however, this point is often chosen at the centroid of the sectioned area.

\*The body's weight is not shown, since it is assumed to be quite small, and therefore negligible compared with the other loads.

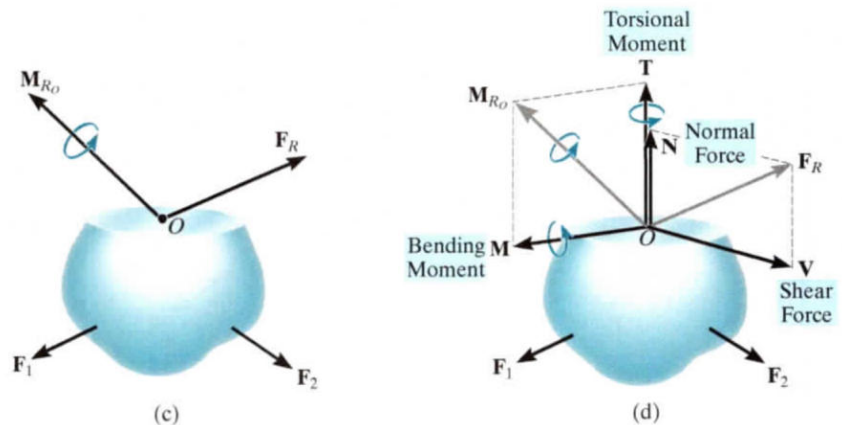


Fig. 1-2 (cont.)



The weight of this sign and the wind loadings acting on it will cause normal and shear forces and bending and torsional moments in the supporting column.

**Three Dimensions.** For later application of the formulas for mechanics of materials, we will consider the components of  $F_R$  and  $M_{R_o}$  acting both normal and tangent to the sectioned area, Fig. 1-2*d*. Four different types of resultant loadings can then be defined as follows:

**Normal force,  $N$ .** This force acts perpendicular to the area. It is developed whenever the external loads tend to push or pull on the two segments of the body.

**Shear force,  $V$ .** The shear force lies in the plane of the area, and it is developed when the external loads tend to cause the two segments of the body to slide over one another.

**Torsional moment or torque,  $T$ .** This effect is developed when the external loads tend to twist one segment of the body with respect to the other about an axis perpendicular to the area.

**Bending moment,  $M$ .** The bending moment is caused by the external loads that tend to bend the body about an axis lying within the plane of the area.

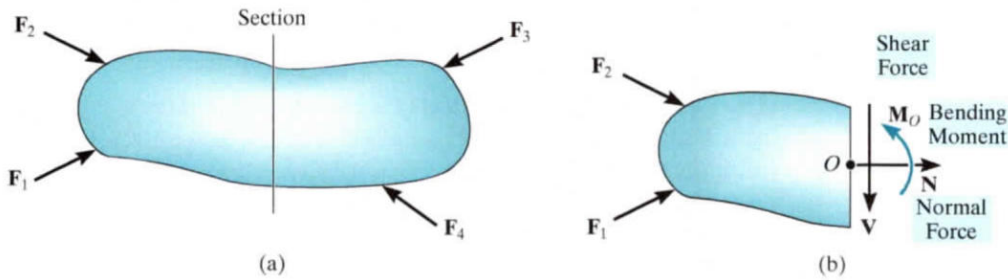


Fig. 1-3

**Coplanar Loadings.** If the body is subjected to a *coplanar system of forces*, Fig. 1-3a, then only normal-force, shear-force, and bending-moment components will exist at the section, Fig. 1-3b.

## IMPORTANT POINTS

- External forces can be applied to a body as *distributed* or *concentrated surface loadings*, or as *body forces* that act throughout the volume of the body.
- Distributed loadings produce a *resultant force* having a *magnitude* equal to the *area* under the distributed load diagram, and act at a *location* that passes through the *centroid* of this area.
- A support produces a *force* in a particular direction on its attached member if it *prevents translation* of the member in that direction, and it produces a *couple moment* on the member if it *prevents rotation*.
- The equations of equilibrium  $\Sigma \mathbf{F} = 0$  and  $\Sigma \mathbf{M} = 0$  must be satisfied in order to prevent a body from translating with accelerated motion and from rotating.
- The method of sections is used to determine the internal resultant loadings acting on the surface of a sectioned body. In general, these resultants consist of a normal force, shear force, torsional moment, and bending moment.