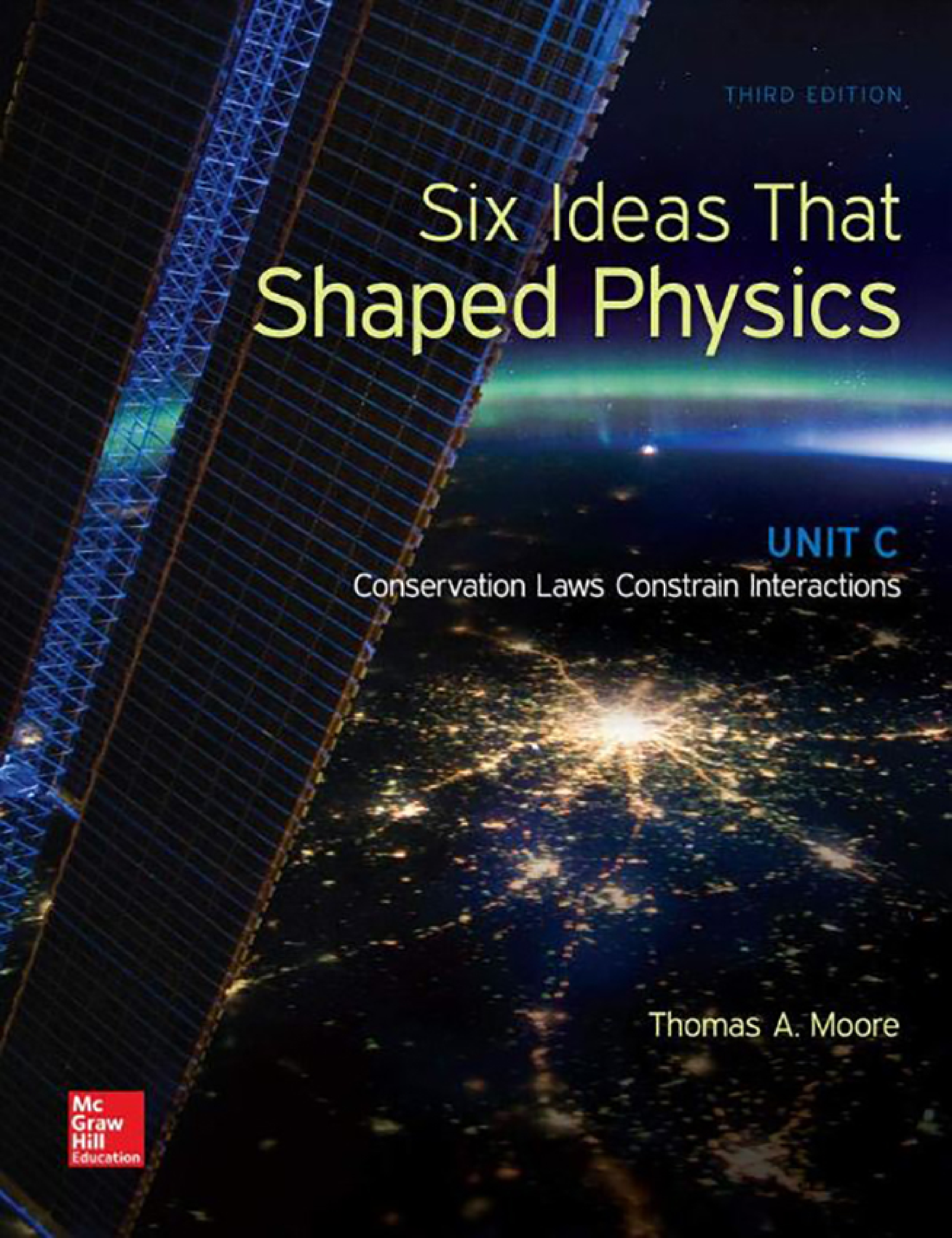


THIRD EDITION

Six Ideas That Shaped Physics

UNIT C

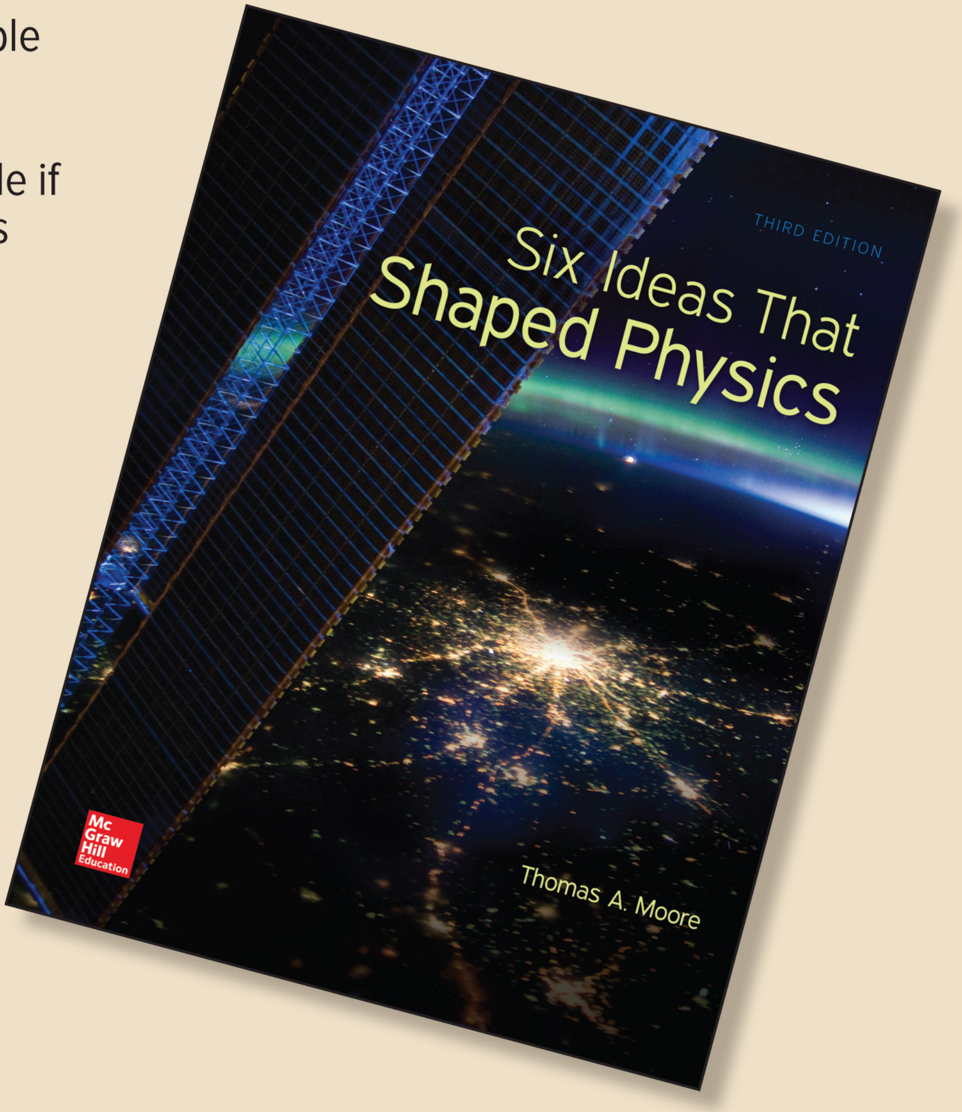
Conservation Laws Constrain Interactions

Thomas A. Moore

McGraw Hill Education

Special Binder-Ready Version

- This loose-leaf alternative will save you money
- Offers a flexible format
- Nonrefundable if shrink-wrap is removed



Mc Graw Hill Education

Six Ideas That Shaped Physics

Third Edition

Thomas A. Moore

McGraw Hill Education

Some Physical Constants

Speed of light	c	$3.00 \times 10^{8}\ \mathrm{m/s}$
Gravitational constant	G	$6.67 \times 10^{-11}\ \mathrm{N{\cdot}m^2/kg^2}$
Coulomb's constant	$1/4\pi\varepsilon_0$	$8.99 \times 10^{9}\ \mathrm{N{\cdot}m^2/C^2}$
Permittivity constant	ε_0	$8.85 \times 10^{-12}\ \mathrm{C^2/(N{\cdot}m^2)}$
Permeability constant	μ_0	$4\pi \times 10^{-7}\ \mathrm{N/A^2}$
Planck's constant	h	$6.63 \times 10^{-34}\ \mathrm{J{\cdot}s}$
Boltzmann's constant	k_B	$1.38 \times 10^{-23}\ \mathrm{J/K}$
Elementary charge	e	$1.602 \times 10^{-19}\ \mathrm{C}$
Electron mass	m_e	$9.11 \times 10^{-31}\ \mathrm{kg}$
Proton mass	m_p	$1.673 \times 10^{-27}\ \mathrm{kg}$
Neutron mass	m_n	$1.675 \times 10^{-27}\ \mathrm{kg}$
Avogadro's number	N_A	6.02×10^{23}

Standard Metric Prefixes
(for powers of 10)

Power	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Commonly Used Physical Data

Gravitational field strength $g = \lvert\vec{g}\rvert$ (near the earth's surface)	$9.80\ \mathrm{N/kg} = 9.80\ \mathrm{m/s^2}$
Mass of the earth M_e	$5.98 \times 10^{24}\ \mathrm{kg}$
Radius of the earth R_e	6380 km (equatorial)
Mass of the sun $M_\odot$	$1.99 \times 10^{30}\ \mathrm{kg}$
Radius of the sun $R_\odot$	696,000 km
Mass of the moon	$7.36 \times 10^{22}\ \mathrm{kg}$
Radius of the moon	1740 km
Distance to the moon	$3.84 \times 10^{8}\ \mathrm{m}$
Distance to the sun	$1.50 \times 10^{11}\ \mathrm{m}$
Density of water†	$1000\ \mathrm{kg/m^3} = 1\ \mathrm{g/cm^3}$
Density of air†	$1.2\ \mathrm{kg/m^3}$
Absolute zero	0 K = −273.15°C = −459.67°F
Freezing point of water‡	273.15 K = 0°C = 32°F
Boiling point of water‡	373.15 K = 100°C = 212°F
Normal atmospheric pressure	101.3 kPa

†At normal atmospheric pressure and 20°C.

‡At normal atmospheric pressure.

Useful Conversion Factors

1 meter = 1 m = 100 cm = 39.4 in = 3.28 ft
1 mile = 1 mi = 1609 m = 1.609 km = 5280 ft
1 inch = 1 in = 2.54 cm
1 light-year = 1 ly = 9.46 Pm = 0.946×10^{16} m
1 minute = 1 min = 60 s
1 hour = 1 h = 60 min = 3600 s
1 day = 1 d = 24 h = 86.4 ks = 86,400 s
1 year = 1 y = 365.25 d = 31.6 Ms = 3.16×10^{7} s
1 newton = 1 N = 1 $\mathrm{kg{\cdot}m/s^2}$ = 0.225 lb
1 joule = 1 J = 1 N·m = 1 $\mathrm{kg{\cdot}m^2/s^2}$ = 0.239 cal
1 watt = 1 W = 1 J/s
1 pascal = 1 Pa = 1 $\mathrm{N/m^2}$ = 1.45×10^{-4} psi
1 kelvin (temperature difference) = 1 K = 1°C = 1.8°F
1 radian = 1 rad = 57.3° = 0.1592 rev
1 revolution = 1 rev = 2π rad = 360°
1 cycle = 2π rad
1 hertz = 1 Hz = 1 cycle/s

1 m/s = 2.24 mi/h = 3.28 ft/s
1 mi/h = 1.61 km/h = 0.447 m/s = 1.47 ft/s
1 liter = 1 l = $(10\ \mathrm{cm})^3 = 10^{-3}\ \mathrm{m^3} = 0.0353\ \mathrm{ft^3}$
1 $\mathrm{ft^3}$ = 1728 $\mathrm{in^3}$ = 0.0283 $\mathrm{m^3}$
1 gallon = 1 gal = 0.00379 $\mathrm{m^3}$ = 3.79 l ≈ 3.8 kg H_2O
Weight of 1-kg object near the earth = 9.8 N = 2.2 lb

1 pound = 1 lb = 4.45 N
1 calorie = energy needed to raise the temperature of 1 g of H_2O by 1 K = 4.186 J
1 horsepower = 1 hp = 746 W
1 pound per square inch = 6895 Pa
1 food calorie = 1 Cal = 1 kcal = 1000 cal = 4186 J
1 electron volt = 1 eV = 1.602×10^{-19} J

$$T = \left(\frac{1\mathrm{K}}{1^\circ\mathrm{C}}\right)(T_{[\mathrm{C}]} + 273.15^\circ\mathrm{C}) \qquad T_{[\mathrm{C}]} = \left(\frac{5^\circ\mathrm{C}}{9^\circ\mathrm{F}}\right)(T_{[\mathrm{F}]} - 32^\circ\mathrm{F})$$

$$T = \left(\frac{5\mathrm{K}}{9^\circ\mathrm{F}}\right)(T_{[\mathrm{F}]} + 459.67^\circ\mathrm{F}) \qquad T_{[\mathrm{F}]} = 32^\circ\mathrm{F} + \left(\frac{9^\circ\mathrm{F}}{5^\circ\mathrm{C}}\right)T_{[\mathrm{C}]}$$

Six Ideas That Shaped Physics

Unit C: Conservation Laws Constrain Interactions

Third Edition

Thomas A. Moore

Mc
Graw
Hill
Education

SIX IDEAS THAT SHAPED PHYSICS, UNIT C:
CONSERVATION LAWS CONSTRAIN INTERACTIONS, THIRD EDITION

Published by McGraw-Hill Education, 2 Penn Plaza, New York, NY 10121. Copyright © 2017 by McGraw-Hill Education. All rights reserved. Printed in the United States of America. Previous editions © 2003, and 1998. No part of this publication may be reproduced or distributed in any form or by any means, or stored in a database or retrieval system, without the prior written consent of McGraw-Hill Education, including, but not limited to, in any network or other electronic storage or transmission, or broadcast for distance learning.

Some ancillaries, including electronic and print components, may not be available to customers outside the United States.

This book is printed on acid-free paper.
1 2 3 4 5 6 7 8 9 0 RMN/RMN 1 0 9 8 7 6

ISBN 978-0-07-351394-2
MHID 0-07-351394-6

Senior Vice President, Products & Markets: *Kurt L. Strand*
Vice President, General Manager, Products & Markets: *Marty Lange*
Vice President, Content Design & Delivery: *Kimberly Meriwether David*
Managing Director: *Thomas Timp*
Brand Manager: *Thomas M. Scaife, Ph.D.*
Product Developer: *Jolynn Kilburg*
Marketing Manager: *Nick McFadden*
Director of Development: *Rose Koos*
Digital Product Developer: *Dan Wallace*
Director, Content Design & Delivery: *Linda Avenarius*
Program Manager: *Faye M. Herrig*
Content Project Managers: *Melissa M. Leick, Tammy Juran, Sandy Schnee*
Design: *Studio Montage, Inc.*
Content Licensing Specialists: *Deanna Dausener*
Cover Image: *NASA*
Compositor: *SPi Global*

Dedication

To All My Family, Near and Far
whose love conserves what is best

All credits appearing on page or at the end of the book are considered to be an extension of the copyright page.

The Internet addresses listed in the text were accurate at the time of publication. The inclusion of a website does not indicate an endorsement by the authors or McGraw-Hill Education, and McGraw-Hill Education does not guarantee the accuracy of the information presented at these sites.

Library of Congress Cataloging-in-Publication Data
Names: Moore, Thomas A. (Thomas Andrew), author.
Title: Six ideas that shaped physics. Unit C, Conservation laws constrain interactions/Thomas A. Moore.
Other titles: Conservation laws constrain interactions
Description: Third edition. | New York, NY : McGraw-Hill Education, [2016] | 2017 | Includes index.
Identifiers: LCCN 2015043352| ISBN 9780073513942 (alk. paper) | ISBN 0073513946 (alk. paper)
Subjects: LCSH: Conservation laws (Physics)—Textbooks.
Classification: LCC QC793.3.C58 M66 2016 | DDC 539.7/54—dc23 LC record available at http://lccn.loc.gov/2015043352

www.mhhe.com

Contents: Unit C
Conservation Laws Constrain Interactions

Chapter C6 84
Conservation of Angular Momentum 84

Chapter C7 100
More About Angular Momentum 100

Chapter C8 118
Conservation of Energy 118

Chapter C9 136
Potential Energy Graphs 136

Chapter C10 154
Work 154

Chapter C11 170
Rotational Energy 170

Chapter C12 186
Thermal Energy 186

Chapter C13 204

Other Forms of Internal Energy 204

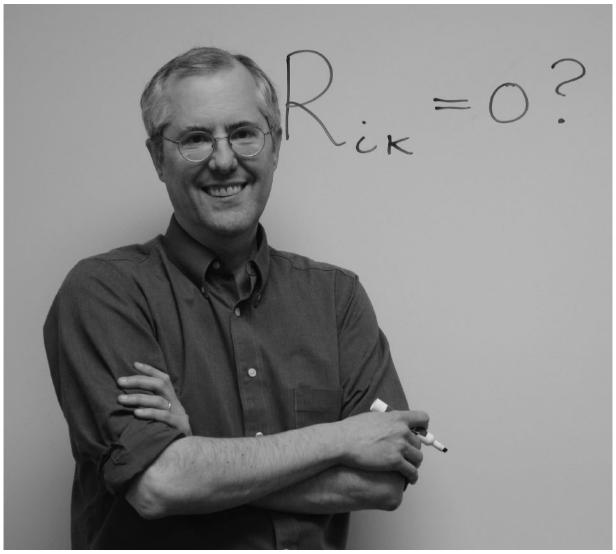
Chapter C14 222

Collisions 222

Appendix CA 240

The Standard Model 240

About the Author



Thomas A. Moore graduated from Carleton College (magna cum laude with Distinction in Physics) in 1976. He won a Danforth Fellowship that year that supported his graduate education at Yale University, where he earned a Ph.D. in 1981. He taught at Carleton College and Luther College before taking his current position at Pomona College in 1987, where he won a Wig Award for Distinguished Teaching in 1991. He served as an active member of the steering committee for the national Introductory University Physics Project (IUPP) from 1987 through 1995. This textbook grew out of a model curriculum that he developed for that project in 1989, which was one of only four selected for further development and testing by IUPP.

He has published a number of articles about astrophysical sources of gravitational waves, detection of gravitational waves, and new approaches to teaching physics, as well as a book on general relativity entitled *A General Relativity Workbook* (University Science Books, 2013). He has also served as a reviewer and as an associate editor for *American Journal of Physics*. He currently lives in Claremont, California, with his wife Joyce, a retired pastor. When he is not teaching, doing research, or writing, he enjoys reading, hiking, calling contradances, and playing Irish traditional fiddle music.

Preface

Introduction

This volume is one of six that together comprise the text materials for *Six Ideas That Shaped Physics*, a unique approach to the two- or three-semester calculus-based introductory physics course. I have designed this curriculum (for which these volumes only serve as the text component) to support an introductory course that combines two elements that rarely appear together: (1) a thoroughly 21st-century perspective on physics (including a great deal of 20th-century physics), and (2) strong support for a student-centered classroom that emphasizes active learning both in and outside of class, even in situations where large-enrollment sections are unavoidable.

This course is based on the premises that innovative metaphors for teaching basic concepts, explicitly instructing students in the processes of constructing physical models, and active learning can help students learn the subject much more effectively. In the course of executing this project, I have completely rethought (from scratch) the presentation of every topic, taking advantage of research into physics education wherever possible. I have done nothing in this text just because "that is the way it has always been done." Moreover, because physics education research has consistently underlined the importance of active learning, I have sought to provide tools for professors (both in the text and online) to make creating a coherent and self-consistent course structure based on a student-centered classroom as easy and practical as possible. All of the materials have been tested, evaluated, and rewritten multiple times. The result is the culmination of more than 25 years of continual testing and revision.

I have not sought to "dumb down" the course to make it more accessible. Rather, my goal has been to help students become *smarter*. I have intentionally set higher-than-usual standards for sophistication in physical thinking, but I have also deployed a wide range of tools and structures that help even average students reach this standard. I don't believe that the mathematical level required by these books is significantly different than that in most university physics texts, but I do ask students to step beyond rote thinking patterns to develop flexible, powerful, conceptual reasoning and model-building skills. My experience and that of other users is that normal students in a wide range of institutional settings can (with appropriate support and practice) meet these standards.

Each of six volumes in the text portion of this course is focused on a single core concept that has been crucial in making physics what it is today. The six volumes and their corresponding ideas are as follows:

Unit C:	**C**onservation laws constrain interactions
Unit N:	The laws of physics are universal (**N**ewtonian mechanics)
Unit R:	The laws of physics are frame-independent (**R**elativity)
Unit E:	**E**lectric and Magnetic Fields are Unified
Unit Q:	Particles behave like waves (**Q**uantum physics)
Unit T:	Some processes are irreversible (**T**hermal physics)

I have listed the units in the order that I *recommend* they be taught, but I have also constructed units R, E, Q, and T to be sufficiently independent so they can be taught in any order after units C and N. (This is why the units are lettered as opposed to numbered.) There are *six* units (as opposed to five or seven) to make it possible to easily divide the course into two semesters, three quarters, or three semesters. This unit organization therefore not only makes it possible to dole out the text in small, easily-handled pieces and provide a great deal of flexibility in fitting the course to a given schedule, but also carries its own important pedagogical message: *Physics is organized hierarchically*, structured around only a handful of core ideas and metaphors.

Another unusual feature of all of the texts is that they have been designed so that each chapter corresponds to what one might handle in a single 50-minute class session at the *maximum possible pace* (as guided by years of experience). Therefore, while one might design a syllabus that goes at a *slower* rate, one should not try to go through *more* than one chapter per 50-minute session (or three chapters in two 70-minute sessions). A few units provide more chapters than you may have time to cover. The preface to such units will tell you what might be cut.

Finally, let me emphasize again that the text materials are just one part of the comprehensive *Six Ideas* curriculum. On the *Six Ideas* website, at

`www.physics.pomona.edu/sixideas/`

you will find a wealth of supporting resources. The most important of these is a detailed instructor's manual that provides guidance (based on *Six Ideas* users' experiences over more than two decades) about how to construct a course at your institution that most effectively teaches students physics. This manual does not provide a one-size-fits-all course plan, but rather exposes the important issues and raises the questions that a professor needs to consider in creating an effective *Six Ideas* course at their particular institution. The site also provides software that allows professors to post selected problem solutions online where their students alone can see them and for a time period that they choose. A number of other computer applets provide experiences that support student learning in important ways. You will also find there example lesson plans, class videos, information about the course philosophy, evidence for its success, and many other resources.

There is a preface for students appearing just before the first chapter of each unit that explains some important features of the text and assumptions behind the course. I recommend that *everyone* read it.

Comments about the Current Edition

My general goals for the current edition have been to correct errors, enhance the layout, improve the presentation in many areas, make the book more flexible, and improve the quality and range of the homework problems as well as significantly increase their number. Users of previous editions will note that I have split the old "Synthetic" homework problem category into "Modeling" and "Derivations" categories. "Modeling" problems now more specifically focus on the process of building physical models, making appropriate approximations, and binding together disparate formulas. "Derivation" problems focus more on supporting or extending derivations presented in the text. I thought it valuable to more clearly separate these categories.

The "Basic Skills" category now includes a number of multipart problems specifically designed for use in the *classroom* to help students practice basic issues. The instructor's manual discusses how to use such problems.

I have also been more careful in providing instructors with more choices about what to cover, making it possible for instructors to omit chapters without a loss of continuity. See the unit-specific part of this preface for more details.

Users of previous editions will also note that I have dropped the menu-like chapter location diagrams, as well as the glossaries and symbol lists that appeared at the end of each volume. I could find no evidence that these were actually helpful to students. Units C and N still instruct students very carefully on how to construct problem solutions that involve translating, modeling, solving, and checking, but examples and problem solutions for the remaining units have been written in a more flexible format that includes these elements implicitly but not so rigidly and explicitly. Students are rather guided in this unit to start recognizing these elements in more generally formatted solutions, something that I think is an important skill.

The only general notation change is that now I use $|\vec{v}|$ exclusively and universally for the magnitude of a vector $\vec{v}$. I still think it is very important to have notation that clearly distinguishes vector magnitudes from other scalars, but the old mag($\vec{v}$) notation is too cumbersome to use exclusively, and mixing it with using just the simple letter has proved confusing. Unit C contains some specific instruction about the notation commonly used in texts by other authors (as well as discussing its problems).

Finally, at the request of *many* students, I now include short answers to selected homework problems at the end of each unit. This will make students happier without (I think) significantly impinging on professors' freedom.

Specific Comments About Unit C

This unit is the foundation on which a *Six Ideas* course rests. The current course structure assumes that unit C is taught first, immediately followed by unit N. Unit C contains core material that will be used in all the other units, as well as providing an introduction to the process of model building that is central to the course.

Why study conservation laws *before* Newtonian mechanics? The most important reasons are as follows: (1) Conservation of "stuff" is a concrete idea that is easy to understand. Beginning with such simple ideas helps build student confidence at the beginning of the course. (2) Using conservation laws does not really require calculus, and so helps students polish their algebra skills before getting involved with calculus. (3) Studying conservation of momentum and angular momentum does require vectors, allowing students to use vectors for several weeks in simple contexts before introducing vector calculus. (4) Conservation laws really are more fundamental than even Newtonian mechanics, so it is good to start the course with concepts that are central and will be used throughout the course.

I did not intuit these benefits at first: the earliest versions of *Six Ideas* presented mechanics in the standard order. Rather, this inversion emerged naturally as a consequence of observations of student learning and some reflection about the course's logical flow.

Inverting the order *can* be a challenge (in both a positive and negative sense) for the student who already has some background in mechanics. Reviewing mechanics from a different perspective can be quite good for such a student because it makes her or him really *think* about the subject again. The instructor can play a key role in helping such students appreciate this and by emphasizing the power and breadth of the conservation law approach and its importance in contemporary physics, as well as celebrating with them the power one gains by being able to approach situations from multiple angles.

The momentum-transfer model of interactions (introduced now in chapter C2) is really what makes it possible to talk about conservation laws without starting with Newton's laws. This model will be a new and challenging idea for almost everyone. Instructors should work carefully with students to give them enough practice with the model to ensure they understand it and can talk about it correctly. The payoff is that when students really grasp this model, it by its very nature helps them avoid many of the standard misconceptions that plague students in introductory courses.

I have substantially revised this unit from the second edition, focusing my attention on the following specific goals.

- I have enlarged the discussion of the model-building process, providing new examples and a more (literally) up-front discussion of tricks and techniques such as unit conversion and dimensional analysis.
- I have reorganized the first few chapters to provide a better logical flow.
- I have substantially rethought how to present expert problem-solving styles. The second edition's experiment with cartoon balloons and interaction diagrams was not very successful with my students. I have replaced this approach with checklists that specify tasks to complete and a more flexible solution style that I think will be easier for students to emulate. Colored comments on many of the example solutions help students see the connection between the solutions and checklists, and there is also some opportunity for students to practice writing the comments themselves, and so become more self-reflective about the process.
- I have brought the two vector conservation laws (momentum and angular momentum) together, which has several pedagogical advantages (including highlighting how these quantities are similar and how they both contrast with energy).
- I have also reorganized the angular momentum chapters so that the basic idea (and what is necessary for later units) appears first and in its own chapter. All the complicated material (involving the cross product) appears in the second chapter, which may be postponed or even omitted.
- I have reorganized the material in the conservation of energy chapters to better even out the pace and improve the logical flow. In particular, I have separated the material on potential energy graphs from material on bonds, latent heat, chemical energy, and nuclear energy (this was all just too much for one chapter).
- Finally, and most importantly, I have reorganized the energy material to be more consistent with the approach that John Jewett outlined in his series of "Energy and the Confused Student" articles in various issues of *The Physics Teacher* in 2008. While I don't completely agree with Jewett on *every* issue, his insights into student difficulties were consistent with what I have observed in the classroom, and I think his approach is superior pedagogically to what I had been doing. This has meant saying farewell to "k-work," which now much more correctly appears as the requirement that conservation of *momentum* imposes on a system. I now also have a complete discussion of "work" that allows a consistent application to deformable systems (including human bodies).

Finally, I have sought to provide more flexibility for instructors. Most of the chapters are crucial and should be discussed in order, but, as noted earlier, chapter C7 on the hard parts of angular momentum may be omitted or delayed, because no other chapter depends on it.

I have also made chapter C14 optional. I think that is very valuable, particularly as a preparation for the last chapters of unit R, but it is not absolutely necessary.

Appreciation

A project of this magnitude cannot be accomplished alone. A list including everyone who has offered important and greatly appreciated help with this project over the past 25 years would be much too long (and such lists appear in the previous editions), so here I will focus for the most part on people who have helped me with this particular edition. First, I would like to thank Tom Bernatowicz and his colleagues at Washington University (particularly Marty Israel and Mairin Hynes) who hosted me for a visit to Washington University where we discussed this edition in detail. Many of my decisions about what was most important in this edition grew out of that visit. Bruce Sherwood and Ruth Chabay always have good ideas to share, and I appreciate their generosity and wisdom. Benjamin Brown and his colleagues at Marquette University have offered some great suggestions as well, and have been working hard on the important task of adapting some *Six Ideas* problems for computer grading.

I'd like to thank Michael Lange at McGraw-Hill for having faith in the *Six Ideas* project and starting the push for this edition, and Thomas Scaife for continuing that push. Eve Lipton and Jolynn Kilburg has been superb at guiding the project at the detail level. Many others at McGraw-Hill, including Melissa Leick, Ramya Thirumavalavan, Kala Ramachandran, David Tietz, and Deanna Dausener, were instrumental in proofreading and producing the printed text. I'd also like to thank Dwight Whitaker of Pomona College and and his Physics 70 students (especially Nathaniel Roy, Eric Cooper, Neel Kumar, Milo Barisof, Sabrina Li, Samuel Yih, Asher Abrahms, Owen Chapman, Mariana Cisneros, Nick Azar, William Lamb, Wuyi Li, Errol Francis, and Cameron Queen) and my students in Physics 71 (especially Alex Hof, Gabrielle Mehta, Gail Gallaher and Jonah Grubb) for helping me track down errors in the manuscript. David Haley and Marilee Oldstone-Moore helped me with several crucial photographs and offered useful feedback. Finally a very special thanks to my wife, Joyce, who sacrificed and supported me and loved me during this long and demanding project. Heartfelt thanks to all!

Thomas A. Moore
Claremont, California

SMARTBOOK®

SmartBook is the first and only adaptive reading experience designed to change the way students read and learn. It creates a personalized reading experience by highlighting the most impactful concepts a student needs to learn at that moment in time. As a student engages with SmartBook, the reading experience continuously adapts by highlighting content based on what the student knows and doesn't know. This ensures that the focus is on the content he or she needs to learn, while simultaneously promoting long-term retention of material. Use SmartBook's real-time reports to quickly identify the concepts that require more attention from individual students–or the entire class. The end result? Students are more engaged with course content, can better prioritize their time, and come to class ready to participate.

Learn Without Limits

Continually evolving, McGraw-Hill Connect® has been redesigned to provide the only true adaptive learning experience delivered within a simple and easy-to-navigate environment, placing students at the very center.

- Performance Analytics – Now available for both instructors and students, easy-to-decipher data illuminates course performance. Students always know how they're doing in class, while instructors can view student and section performance at-a-glance.
- Mobile – Available on tablets, students can now access assignments, quizzes, and results on-the-go, while instructors can assess student and section performance anytime, anywhere.
- Personalized Learning – Squeezing the most out of study time, the adaptive engine in Connect creates a highly personalized learning path for each student by identifying areas of weakness, and surfacing learning resources to assist in the moment of need. This seamless integration of reading, practice, and assessment, ensures that the focus is on the most important content for that individual student at that specific time, while promoting long-term retention of the material.

Introduction for Students

Introduction

Welcome to *Six Ideas That Shaped Physics!* This text has a number of features that may be different from science texts you may have encountered previously. This section describes those features and how to use them effectively.

Why Is This Text Different?

Why *active learning* is crucial

Research into physics education consistently shows that people learn physics most effectively through *activities* where they practice applying physical reasoning and model-building skills in realistic situations. This is because physics is not a body of facts to absorb, but rather a set of thinking skills acquired through practice. You cannot learn such skills by listening to factual lectures any more than you can learn to play the piano by listening to concerts!

This text, therefore, has been designed to support *active learning* both inside and outside the classroom. It does this by providing (1) resources for various kinds of learning activities, (2) features that encourage active reading, and (3) features that make it as easy as possible to use the text (as opposed to lectures) as the primary source of information, so that you can spend class time doing activities that will actually help you learn.

The Text as Primary Source

Features that help you use the text as the primary source of information

To serve the last goal, I have adopted a conversational style that I hope you will find easy to read, and have tried to be concise without being too terse.

Certain text features help you keep track of the big picture. One of the key aspects of physics is that the concepts are organized *hierarchically*: some are more fundamental than others. This text is organized into six units, each of which explores the implications of a single deep idea that has shaped physics. Each unit's front cover states this **core idea** as part of the unit's title.

A two-page **chapter overview** provides a compact summary of that chapter's contents to give you the big picture before you get into the details and later when you review. **Sidebars** in the margins help clarify the purpose of sections of the main text at the subpage level and can help you quickly locate items later. I have highlighted technical terms in bold type (like **this**) when they first appear: their definitions usually appear nearby.

A physics **formula** consists of both a mathematical equation and a *conceptual frame* that gives the equation physical meaning. The most important formulas in this book (typically, those that might be relevant outside the current chapter) appear in **formula boxes**, which state the equation, its ***purpose*** (which describes the formula's meaning), a description of any ***limitations*** on the formula's applicability, and (optionally) some other useful ***notes***. Treat everything in a box as a unit to be remembered and used together.

Active Reading

What is *active reading*?

Just as passively listening to a lecture does not help you really learn what you need to know about physics, you will not learn what you need by simply scanning your eyes over the page. **Active reading** is a crucial study skill for all kinds of technical literature. An active reader stops to pose internal questions such as these: Does this make sense? Is this consistent with my experience? Do I see how I might be able to use this idea? This text provides two important tools to make this process easier.

Features that support developing the habit of active reading

Use the **wide margins** to (1) record *questions* that arise as you read (so you can be sure to get them answered) and the *answers* you eventually receive, (2) flag important passages, (3) fill in missing mathematical steps, and (4) record insights. Writing in the margins will help keep you actively engaged as you read and supplement the sidebars when you review.

Each chapter contains three or four **in-text exercises**, which prompt you to develop the habit of *thinking* as you read (and also give you a break!). These exercises sometimes prompt you to fill in a crucial mathematical detail but often test whether you can *apply* what you are reading to realistic situations. When you encounter such an exercise, stop and try to work it out. When you are done (or after about 5 minutes or so), look at the answers at the end of the chapter for some immediate feedback. Doing these exercises is one of the more important things you can do to become an active reader.

SmartBook (TM) further supports active reading by continuously measuring what a student knows and presenting questions to help keep students engaged while acquiring new knowledge and reinforcing prior learning.

Class Activities and Homework

Read the text BEFORE class!

This book's *entire purpose* is to give you the background you need to do the kinds of *practice* activities (both in class and as homework) that you need to genuinely learn the material. *It is therefore ESSENTIAL that you read every assignment BEFORE you come to class*. This is *crucial* in a course based on this text (and probably more so than in previous science classes you have taken).

Types of practice activities provided in the text

The homework problems at the end of each chapter provide for different kinds of practice experiences. **Two-minute problems** are short conceptual problems that provide practice in extracting the implications of what you have read. **Basic Skills** problems offer practice in straightforward applications of important formulas. Both can serve as the basis for classroom activities: the letters on the book's back cover help you communicate the answer to a two-minute problem to your professor (simply point to the letter!). **Modeling** problems give you practice in constructing coherent mental models of physical situations, and usually require combining several formulas to get an answer. **Derivation** problems give you practice in mathematically extracting useful consequences of formulas. **Rich-context** problems are like modeling problems, but with elements that make them more like realistic questions that you might actually encounter in life or work. They are especially suitable for collaborative work. **Advanced** problems challenge advanced students with questions that involve more subtle reasoning and/or difficult math.

Note that this text contains perhaps fewer examples than you would like. This is because the goal is to teach you to *flexibly reason from basic principles*, not slavishly copy examples. You may find this hard at first, but real life does not present its puzzles neatly wrapped up as textbook examples. With practice, you will find your power to deal successfully with realistic, practical problems will grow until you yourself are astonished at how what had seemed impossible is now easy. *But it does take practice*, so work hard and be hopeful!

C1

The Art of Model Building

Chapter Overview

Section C1.1: The Nature of Science

One of the main goals of science is the development of imaginative **conceptual models** of physical reality. A model deliberately simplifies a complex reality in such a way that it captures its essence and helps us think more clearly about it. *This text's main purpose is to teach you the art of scientific model building,* by helping you not only understand and appreciate the grand models we call **theories** but also practice building the small-scale models one needs to apply a theory in a given situation.

Science is an unusually effective process for generating powerful models of reality that involves four crucial elements coming together:

1. A sufficiently large community of scholars, who share
2. A commitment to logical consistency as an essential feature of all models,
3. An agreement to use **reproducible experiments** to test models, and
4. A grand theory rich enough to provide a solid foundation for research.

In the case of physics, the Greek philosophical tradition created a community that valued logical reasoning. Early Renaissance thinkers championed the value of reproducible experiments as being crucial for testing models. But physics was not really launched until 1687, when Newton provided a theory of mechanics grand and compelling enough to unify the community and provide a solid context for research.

Section C1.2: The Development and Structure of Physics

Since the days of Newton, physicists have sought to create models able to embrace originally distinct areas of study and thus cover broader ranges of physical phenomena. The current conceptual structure of physics, illustrated in figure C1.1, rests on *two* grand theories: **general relativity** (GR) and the **Standard Model** (SM) of particle physics. In practice, though, physicists almost always use five simpler theories (which are approximations valid in various limited contexts): **newtonian mechanics**, **special relativity**, **electromagnetic field theory**, **quantum mechanics**, and **statistical mechanics**. This text focuses on these five models.

Physicists have recently come to appreciate the role that **symmetries** play in physics. Both GR and the SM acknowledge (as almost any imaginable theory must) certain symmetries (such as the time and position independence of physical laws) that give rise to **conservation laws** (such as the laws of conservation of energy and momentum). Such laws have a validity beyond the specific theories currently in vogue. Indeed, GR and the SM themselves are based on new, nonobvious symmetries.

Section C1.3: A Model-Building Example

To apply a grand theoretical model to any actual physical situation of interest, a scientist must construct an idealized model that simplifies the situation, bringing its essence into focus in such a way that one can easily connect it to the grand model. This course is designed to help you *practice* this process, which is really the only way to learn how to do it. This section illustrates what is involved in an example situation.

In the process, the section describes some useful tricks that can help you simplify situations and think about which simplifications are appropriate:

1. Lines or rays from a very distant point are nearly parallel.
2. The length of a gentle curve between two points is almost the same as that of a straight line between those points.
3. The fractional uncertainty of a result calculated by multiplication or division from uncertain quantities is roughly equal to that of the most uncertain quantity involved. The same is true for the sine or tangent of small angles.

Part of the art of model building is to develop a bag of such tricks that you can pull out when helpful. The only real way to learn these tricks is by *practice*, and also by making mistakes that you learn to correct. So be bold and learn from your mistakes!

Solutions to most physics problems involve three different sections:

1. A ***model*** section that describes the simplifications one makes to the situation
2. A ***math*** section where one does the mathematics implied by the model
3. A ***check*** section where one decides whether the result makes sense

Your earlier experience with more trivial problems may lead you to neglect the ***model*** and ***check*** sections, but I strongly recommend you do not. The ***model*** section is particularly important in this course. A good and sufficiently well-labeled diagram is often the core of a sufficient model for problem solutions you prepare.

Section C1.4: Trick Bag: Unit Awareness

One of the most powerful tricks you can put in your bag is *being aware of units*. Units give meaning to quantities and are essential for correctly communicating that meaning to others. Being constantly aware of units (even when working with symbolic equations) is one of the best and easiest ways to spot mistakes in your work.

Here is a list of the things you should know to increase your unit awareness:

1. Know the basic and derived SI units and SI prefixes (see the inside front cover).
2. Know and/or refer to the SI unit benchmarks in figure C1.2.
3. Know that the units on both sides of an equation must match.
4. Know that you cannot add or subtract quantities with different units, but you can multiply or divide them.
5. Know that you should be aware of units even in symbolic equations.
6. Know that math functions take unitless arguments and yield unitless results.

Section C1.5: Trick Bag: Unit Conversions

In most physics problem solutions, you will need to convert units. My preferred technique for doing this is the **unit operator** method, where you convert unit equalities such as 1 mile = 1609 meters into a ratio equal to 1 such as 1 = (1 mi/1609 m) or 1 = (1609 m/1 mi). Since anything can be multiplied by 1 without changing it, you can multiply any quantity by such a unit operator and cancel units top and bottom (as if they were algebraic symbols) until only the units you want are left over. For example: $23\text{ mi} = (23\,\cancel{\text{mi}})(1609\text{ m}/1\,\cancel{\text{mi}}) = 37{,}000\text{ m}$. This method is *foolproof* as long as you pay attention to unit consistency and to canceling units correctly.

Section C1.6: Trick Bag: Dimensional Analysis

Dimensional analysis is a surprisingly powerful trick that often yields good estimates of physics formulas and/or quantities without requiring anything more than the most basic knowledge of a situation. As such, it often represents the simplest model you can construct of a given situation.

This trick takes advantage of the facts that (1) units must agree on both sides of any equation, (2) that most formulas in physics are simple power laws, and (3) that most unitless constants appearing in such formulas are within a factor of 10 or so of one. The steps in applying dimensional analysis to a situation are as follows:

1. Decide what quantities your desired value might depend on.
2. Assume that these quantities appear in a power law formula (e.g., $Q = KA^mB^n$).
3. Find the powers by requiring units to be consistent on both sides of the formula.
4. Assume that the unitless constant K in front of the equation is 1.

C1.1 The Nature of Science

By our nature, we humans strive to discern order in the cosmos and love to tell stories that use ideas from our collective experience to "explain" what we see. Science stands firmly in this ancient tradition: stories about how the gods guide the planets around the sky and modern stories about how spacetime curvature does the same have much in common. What distinguishes science from the rest of the human storytelling tradition are (1) the *types* of stories scientists tell, (2) the *process* that they use for developing and sifting these stories, and (3) the *predictive success* enjoyed by the surviving stories.

Scientists express their stories in the form of **conceptual models**, which bear a similar relation to the real world as a model airplane does to a real jet. A good scientific model captures a phenomenon's essence while being small and simple enough for a human mind to grasp. Models are *essential* because reality is too complicated to understand fully; models distill complex phenomena into bite-sized chunks that finite minds can digest. Framing a model is less an act of discovery than of imagination: a good model is a compelling *story* about reality that creatively ignores just the right amount of complexity.

Model building occurs at all levels of science

Model-making in science happens at all levels. **Theories**—grand models embracing a huge range of phenomena—are for science what great novels are for literature: soaring works of imagination that we study and celebrate for their insight. But applying such a grand model to a real-life situation requires building a smaller model of the situation itself, simplifying the situation and making appropriate approximations to help us connect it to the grand model. Scientists do this second kind of model-making daily, and one of the main goals of this course is to help you learn that art.

Because models are necessarily and consciously simpler than reality, all have limits: the full "truth" about any phenomenon can never be told. Pushing any model far enough eventually exposes its inadequacies. Even so, one *can* distinguish better from poorer models. Better models are more logical, more predictive in a broader range of cases, more elegantly constructed, and more productive in generating further research than poorer ones are.

Science is really a *process* for building, evaluating, and refining models, one that (since its beginnings in the 1600s) has proved to be an astonishingly prolific producer of powerful and trustworthy models. It owes part of its success to its focus on the *natural* world, whose orderly behavior at many levels makes finding and testing models easier than in the world of human culture.

Scholars of the philosophy and history of science suggest that a discipline becomes a science only when the following four elements come together:

1. A sufficiently large *community* of scholars, who share
2. A commitment to *logical consistency* as an essential feature of all models,
3. An agreement to use **reproducible experiments** to test models, and
4. A *grand theory* rich enough to provide a solid foundation for research.

How physics became a science

In the case of physics, the Greek philosophical tradition founded a community of scholars who appreciated the power of logical reasoning: indeed, this community found logic's power so liberating that it long imagined pure logic to be *sufficient* for knowing. The idea of using experiments to *test* one's logic and assumptions was not even fully expressed until the 13th century, and was not recognized as *necessary* until the 17th. Eventually, though, the community recognized that the human desire to order experience is so strong that the core challenge facing a thinker is to distinguish real order from merely imagined patterns. *Reproducible experiments* make what would otherwise be individual experience available to a wider community, anchoring models more firmly to reality. Galileo Galilei (1564–1642) was a great champion of this approach. His use of the newly invented telescope to display features of heavenly bodies unanticipated by models of the time underlined to his peers the inadequacy of pure reason and the importance of observation.

A prescientific community lacking a grand theory, however, tends to fragment into schools, each championing its own theory. Rapid progress is thwarted because each school sees any collected data through the lens of its cherished model, making arguments virtually impossible to resolve. This was the situation in physics during most of the 1600s. However, in 1687 Isaac Newton published an ingenious model of physics broad enough to embrace both terrestrial and celestial phenomena. His grand theory captured the imagination of the entire physics community, which turned away from arguing about partial models and toward working *together* to refine, test, and extend Newton's basic theory, confident that it would be shown to be universally true and valid. At this moment, physics became a science.

The ironic paradox at the heart of science

The unified community now made rapid progress in constructing powerful subordinate models that greatly extended the reach of Newton's grand vision, feeding the Industrial Revolution along the way. Ironically, the community that strove energetically to extend Newton's model universally eventually amassed evidence proving it *incomplete*! Only a community *devoted* to a theory can collect the kind of detailed and careful evidence necessary to expose its inadequacies, and thus move on to better theories. This irony is the engine that drives science forward.

C1.2 The Development and Structure of Physics

The history of physics since Newton

Unification of apparently distinct models has been an important theme in the development of physics since Newton's theory unified terrestrial and celestial physics. In the 1800s, work on electricity, magnetism, and light (initially described by distinct partial models) culminated in an "electromagnetic field model" embracing them all, and physicists found how to subsume thermal phenomena into Newton's model. This process was going so well in the late 1800s that the physicist Lord Kelvin famously claimed that there was probably little left to learn about physics!

In the early 1900s, though, physicists began to see that certain experimental results were simply incompatible with Newton's framework. After what amounted to a period of revolution, the community demoted Newton's theory and coalesced around two new grand theories—**general relativity** (1915) and **quantum mechanics** (1926)—which embraced the new results but yielded the same results as Newton's theory in the appropriate limits.

In the 1950s, physicists were able to unify quantum mechanics, electromagnetic field theory, and **special relativity** (the nongravitational part of general relativity) to create *quantum electrodynamics* (QED), the first example of a **relativistic quantum field theory**. In the 1970s, physicists extended this model to create relativistic quantum field theories to describe two new (subatomic-scale) interactions discovered in previous decades and integrated them with QED into a coherent theory of subatomic particle physics called the **Standard Model**. This model has been quite successful, predicting new phenomena and particles that have been subsequently observed. The model's latest triumph was the discovery of the predicted "Higgs boson" in 2012.

The current structure of physics

Currently, **general relativity**, which covers gravity and other physical phenomena at distance scales larger than molecules, and the **Standard Model**, which works in principle at all distance scales but does not and cannot cover gravity, stand as the squabbling twin grand theories of physics. Though no known experimental result defies explanation by one or the other, physicists

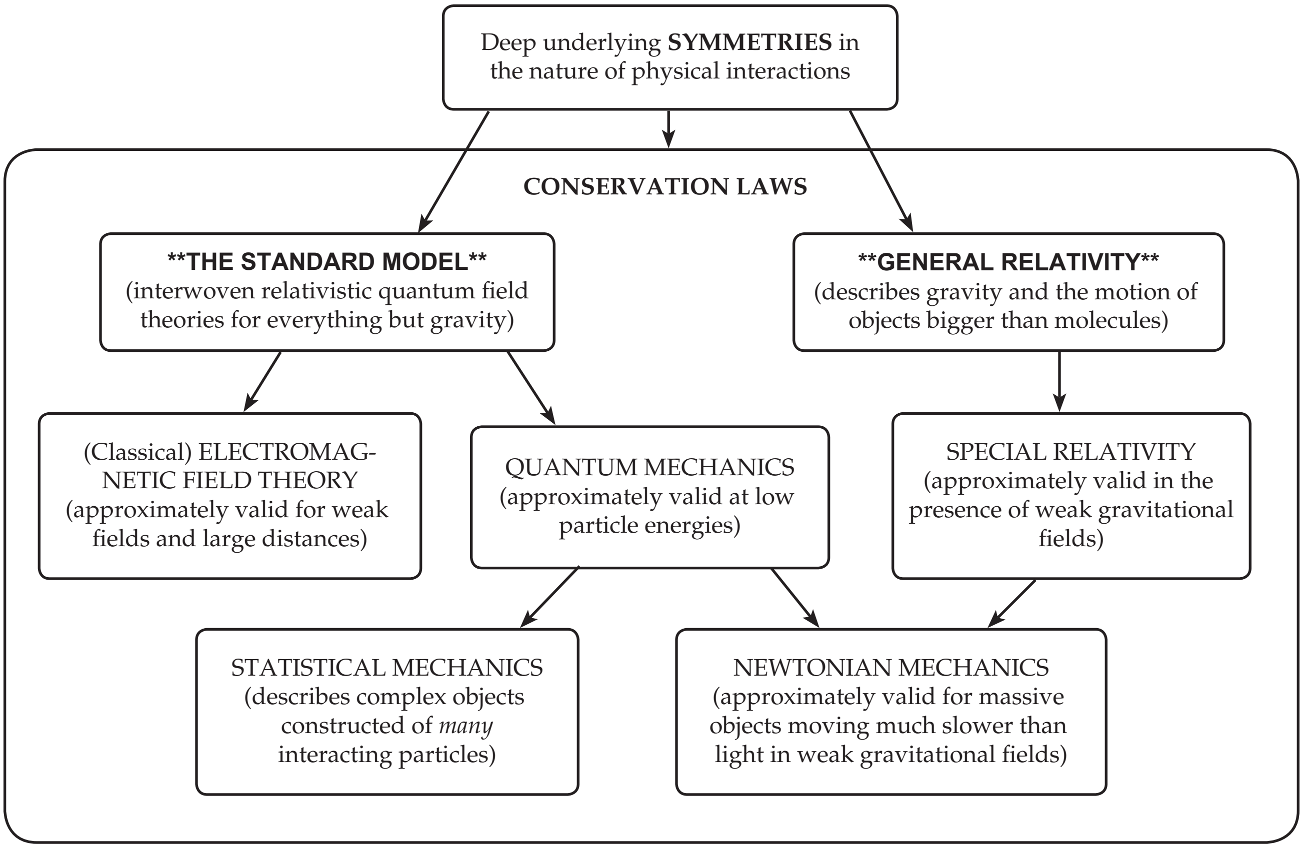


Figure C1.1
The current grand theories of physics (starred) and the five approximate models more often used in practice.

are dissatisfied with each theory (for different reasons) and especially distressed that we need two deeply incompatible theories instead of one. While many unifying models have been proposed (**string theory** and **loop quantum gravity** are examples), these models lack both the level of development and the firm experimental basis to inspire general acceptance. The physics community is thus presently in the curious position of being devoted to two grand theories we already *know* to be wrong (or at best incomplete).

In *practice*, however, physicists rarely use either to explain any but the most exotic phenomena. Instead, they use one of five simpler theories: **newtonian mechanics**, **special relativity**, **electromagnetic field theory**, **quantum mechanics**, and **statistical mechanics**. Each has a more limited range of applicability than the two grand theories, but is typically *much* easier to use within that range. These theories, their limitations, and their relationships to the grand theories and each other are illustrated in figure C1.1.

The importance of symmetries in physics

This diagram also emphasizes the importance of **symmetry principles** in physics. Early in the 1900s, mathematician Emmy Noether showed that, given plausible assumptions about the form that physical laws must have, a symmetry principle stating that “the laws of physics are unaffected if you do such-and-such” automatically implies an associated **conservation law**. For example, the *time-independence* of the laws of physics (whatever those laws might be) implies that a quantity that we call **energy** is *conserved* (that is, does not change in time) in an isolated set of objects obeying those laws.

Conservation laws, therefore, stand independently and behind the particular models of physics, as figure C1.1 illustrates. For example, conservation of energy is a feature of newtonian mechanics, electromagnetic field theory, special relativity, quantum mechanics, *and* statistical mechanics because *all* these theories involve physical laws that (1) have forms consistent with Noether's theorem and (2) are assumed to be time independent. Each theory has a different way of *defining* energy, but all agree that it is *conserved*.

Symmetry principles are also important because both our current grand models of physics (the Standard Model and general relativity) propose and unravel the consequences of new and nonobvious symmetry principles. The section of this text on special relativity illustrates this by displaying how relativity's mind-blowing features are in fact simple logical consequences of the symmetry principle that "the laws of physics are unaffected by one's state of (uniform) motion." Linking other symmetry principles with their consequences is (unfortunately) not quite so simple (and is beyond the level of this course), but is not qualitatively different.

Why one must learn the five simpler theories first

Now, given the structure of physics illustrated in figure C1.1, it might seem logical to begin studying physics by starting with the two fundamental theories (or even the symmetry principles) and then working downward to the five approximate theories. However, this is impractical because the fundamental theories, in spite of their awesome breadth and beauty, are (1) very sophisticated mathematically and conceptually, (2) unnecessarily complicated to use in most contexts, and (3) necessarily expressed using the language and concepts of the five simpler theories. One must therefore start by learning those simpler theories. The other five volumes of this textbook will provide you with a very basic introduction to all five of these simpler theories, as well as exploring many of the supporting models that help broaden their range. This unit begins the process by looking at the conservation laws (in the context of newtonian mechanics) that underlie all these theories.

In the remainder of this chapter, though, we will explore the model-building process in more detail and develop some general tools that help us avoid errors and maximize what we can gain from even limited knowledge.

C1.3 A Model-Building Example

One learns the art of model building through *practice*

This book is designed partly to teach you the kind of creative model-building that working scientists do daily. The model-building process cannot be reduced to formulaic procedures that one can follow like a recipe. It is an art that requires knowledge, intelligence, creativity, and most of all, *practice.* You can no more learn this art simply by reading books or attending lectures than you can learn to play the piano simply by attending concerts.

An example that illustrates the model-building process

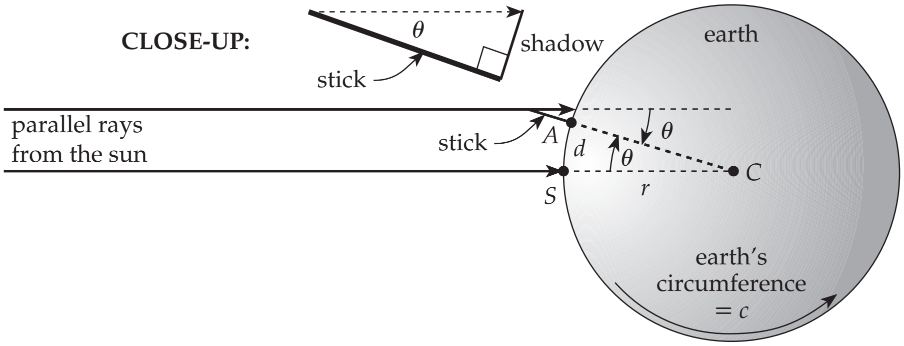
So let's practice! The exercise below poses a simple question you can answer using some basic trigonometry and geometry, grade-school science, and a bit of creative model building. Spend at least 10 minutes but no more than 15 minutes trying to answer the question before turning the page.

Exercise C1X.1

About 240 B.C.E., Eratosthenes made the first good estimate of the earth's size as follows. Caravan travelers told him that in the village of Syene, one could see the sun reflected in a deep well at noon on the summer solstice, meaning that it was directly overhead. Eratosthenes noted that at the same time on the same day in Alexandria (5000 Greek stadia to the north, as estimated by camel travel time), a vertical stick cast a shadow about 1/8 of its length. What is the earth's radius in stadia? (*Hint*: Draw a picture. In 240 B.C.E., the Greeks knew that the earth was spherical and the sun was very far away.)

Example C1.1

Model If the sun is sufficiently far from the earth, light rays traveling from the sun to Alexandria (point A) and Syene (point S) will be almost parallel. Let's assume they are *exactly* parallel and that the earth is perfectly spherical. The figure below shows a cross-sectional view of the situation from the east.



Because alternate interior angles are equal, the angle θ between the stick and the sun's rays at Alexandria is the same as the angle θ between lines AC and SC, where C is the earth's center. The stick is vertical by assumption, so it is perpendicular to its shadow. Thus, the stick, the shadow, and the ray passing the stick's end form a right triangle (see the "close-up" above). If the shadow is 1/8 the stick's length, then $\tan\theta = 1/8$, so $\theta = \tan^{-1}(1/8) = 7.1°$. The distance d between Alexandria and Syene is to the earth's circumference c as θ is to 360°, so $c/d = 360°/\theta$. Note also that $c = 2\pi r$, where r is the earth's radius.

Math Therefore

$$r = \frac{c}{2\pi} = \frac{d}{2\pi}\left(\frac{360°}{\theta}\right) = \frac{(5000\text{ stadia})(360°)}{2\pi(7.1°)} = 40{,}000\text{ stadia} \qquad \text{(C1.1)}$$

Check The exact length of Eratosthene's stadion is historically ambiguous, but if he meant the "itinerary stadion" (the one used for road trips), then 1 stadion = 0.157 km and r is 6300 km, pretty close to the modern value.

If you got something like this, congratulations! If you had trouble getting a useful result in 15 minutes, that's normal. Doing a moderately realistic problem like this is *hard,* not usually because the math or concepts are hard (both are pretty basic here), but because *constructing the model* is hard. How does one know what approximations to make? How does one create a schematic diagram of a situation (like the one shown) that usefully exposes its essential features? How do you frame things so that the mathematics is simple?

You may even be annoyed with my solution: "Well," you might say, "if I had known it was acceptable to make the false assumption that rays from the sun are parallel, then the solution would have been easy!" That is precisely the point! It is not only acceptable but also usually *necessary* to make simplifications *to solve a problem at all*. The trick is simplifying *just enough* to make the problem tractable without making the result uselessly crude. There are no "correct" answers in such a case, only poorer and better models that yield poorer or better results (and if a poor result is the best one can do, it is still better than nothing!). This is where the creativity and artistry comes in. My goal is to help you learn to simplify (that is, to be *productively and creatively* lazy) imaginatively, boldly, and exuberantly!

The simplifications and assumptions involved in example C1.1's model

With this in mind, let's examine more closely the simplifications and assumptions behind the model in example C1.1. The solution assumes that the sun is sufficiently distant that light rays from it are parallel at the earth: if this is not so, the two angles marked θ in the diagram are not equal. No one knew the distance to the sun in Eratosthenes's time, so his assumption was quite bold, but we now know that two rays from a single point on the sun that arrive at Syene and Alexandria, respectively, are *not* parallel, but actually make an angle of about 0.00035° with each other. Solving this problem "more correctly" by taking this into account yields an r that is smaller by about 0.005%.

However, this is truly insignificant compared with other simplifications we are making. The sun's angular diameter when viewed from the earth is about 0.5°, so there is not just one ray that grazes the top of the stick and connects it with the shadow on the ground, but rather a bundle of rays that could make angles with each other of as much as 0.53°. This means that the end of the stick's shadow will be blurred, making its length and thus the angle θ uncertain by about ±0.26°. Also the hills and valleys between Syene and Alexandria make the road distance d longer than the distance that would be measured on a perfect sphere. Moreover, Alexandria is not due north of Syene, as the drawing assumes it is. The earth is also not exactly spherical (its polar radius is smaller than its equatorial radius by about 11 km).

I could state yet more subtle assumptions, but I think you get the point. Reality is complicated, and the model simply ignores those complications.

The "weakest link rule" for uncertain quantities

Now, it turns out if you multiply or divide uncertain or erroneous quantities, the result has (roughly) the same *percent* uncertainty as the most uncertain of the quantities. This **weakest-link rule** is also (roughly) true for tangents or sines of small angles. (Check it out for yourself: see problem C1D.1.) In this case, the uncertainty in θ is roughly ±4% (±0.26°/7.1°) because of the angular width of the sun, and the uncertainty in the distance d is likely to be more than ±10%, since it is determined by camel travel time! The weakest-link rule implies, therefore, that we are not going to know the radius of the earth to better than about ±10% no matter how good our model is. Making a *far* more complicated model to correct the approximations described above is not going to make the slightest bit of practical difference: we are simply not given good enough information to calculate the earth's radius more precisely. The problem (as stated) therefore does not *deserve* a better model!

The art of "good enough"

Part of the art of model building is knowing when a model is "good enough." Eratosthenes's model was not merely "good enough;" it was pure genius at the time, since no other method of determining the earth's radius was remotely as good. (Sometimes even a crude result is a big step forward!) One learns the art of "good enough" mostly by practice. Indeed, I hope this course will give you (among other things) a bag of useful tricks that are often "good enough." Treating lines from a distant point as parallel is one such trick. The weakest-link rule about the uncertainty of multiplied or divided quantities is another. Practice with tricks like these puts them into your bag.

Good diagrams are essential

Another important trick is recognizing the importance of a good diagram. Drawing (and carefully labeling) the drawing in example C1.1 was probably *the single most important thing* I did to solve the problem. Most of the "Model" in my solution merely restates the diagram verbally. A good diagram is *often* the most important trick for solving a physics problem.

The three sections of almost any physics problem solution

Indeed, solutions to all but the most trivial problems will involve the three sections appearing in the example solution: a ***model*** section where one draws a schematic diagram and/or discusses approximations and assumptions, a ***math*** section where one does the mathematics implied by the model to solve for the desired result, and a ***check*** section where one checks the result to see if it makes sense.