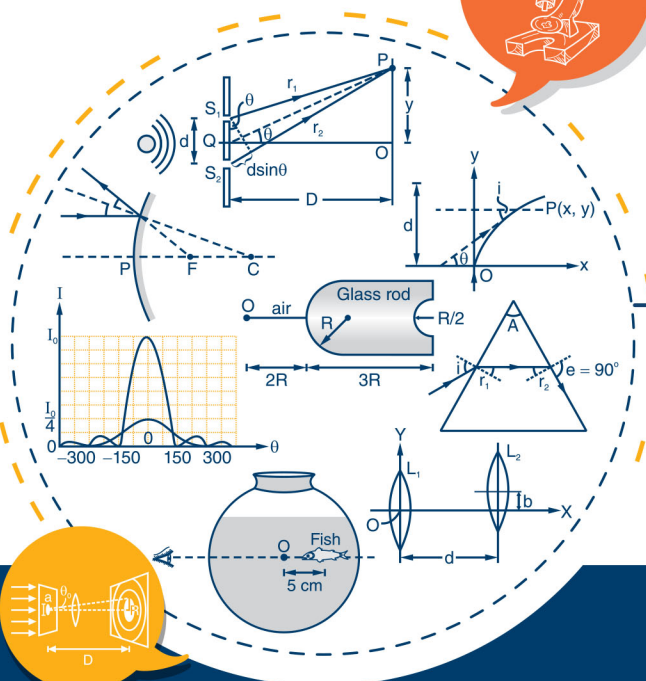


# 3

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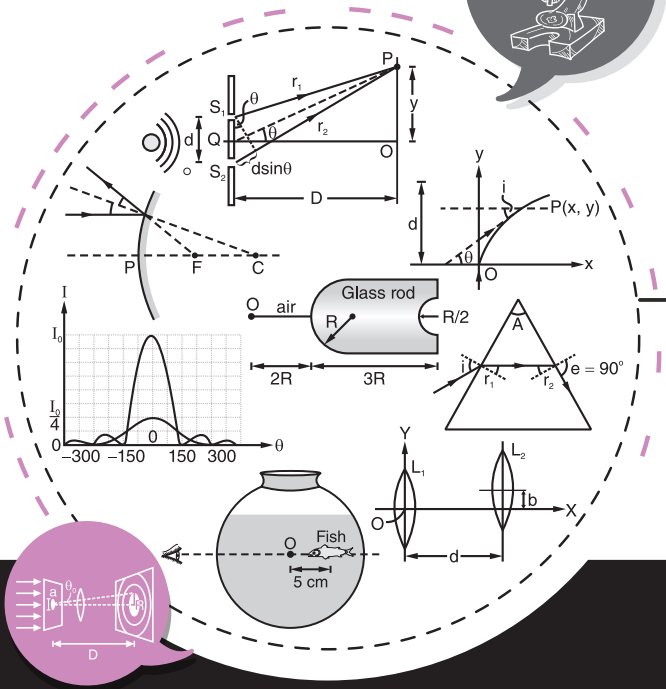
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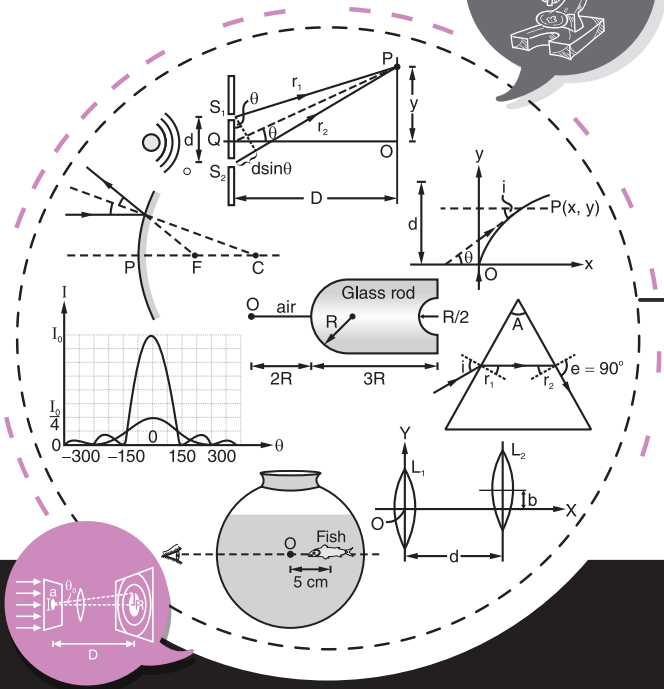
THIRD EDITION



# JEE

## ADVANCED

# PHYSICS



# Optics

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## HINTS AND EXPLANATIONS

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# CHAPTER INSIGHT

## Learning Objectives

Help the students set an aim to achieve the major take-aways from a particular chapter.



## Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

- |  |  |
|--|--|
| (a) Reflection for plane and curved surfaces (i.e. for plane and curved mirrors) | (d) Lens   |
| (b) Refraction for plane surfaces (i.e. for glass slab and prism)                | (e) Lens Makers Formula                          |
| (c) Refraction for curved surfaces   | (f) Human eye                                    |
|  | (g) Defects in human eye and optical instruments |

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the

## Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

- |  |   |
|--|---|
| (a) Wave nature of light                                       | (e) Diffraction phenomenon              |
| (b) Huygen's Principle   | (f) Resolving power                     |
| (c) Interference   | (g) Fresnel's distance and Polarisation |
| (d) Young's Double Slit Experiment (along with its variations) |   |

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

### ILLUSTRATION 24

A ray of light falls on a glass plate of refractive index  $n = \sqrt{3}$ . What is the angle of incidence of the ray if the angle between the reflected and refracted rays is  $90^\circ$ ?

#### SOLUTION

According to Snell's Law

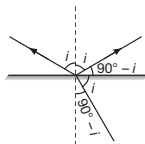
$$n = \frac{\sin i}{\sin r}$$

Since  $i + r = 90^\circ$

$$\Rightarrow r = 90^\circ - i$$

$$\Rightarrow \sqrt{3} = \frac{\sin i}{\sin(90^\circ - i)} = \tan i$$

$$\Rightarrow i = \tan^{-1}(\sqrt{3}) = 60^\circ$$



### ILLUSTRATION 25

A ray of light passes through a medium whose refractive index varies with distance as  $n = n_0 \left(1 + \frac{x}{2a}\right)$ . If ray enters the medium parallel to  $x$ -axis, what will be the time taken for ray to travel between  $x = 0$  and  $x = a$ ?

#### SOLUTION

Since, we know that  $\mu = \frac{c}{v}$

$$\Rightarrow \frac{A_1}{A_2} = 2$$

### ILLUSTRATION 4

The intensity of the light coming from one of the slits in YDSE is double the intensity from the other slit. Find the ratio of the maximum intensity to the minimum intensity in the interference fringe pattern observed.

#### SOLUTION

Since, we know that

$$\frac{I_{\max}}{I_{\min}} = \left( \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} \right)^2$$

According to the problem, we have

$$I_1 = 2I_0 \text{ and } I_2 = I_0$$

$$\Rightarrow \frac{I_{\max}}{I_{\min}} = \left( \frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right)^2 = 34$$

### ILLUSTRATION 5

In a Young's double-slit experiment the distance between the slits is 1 mm and the distance of the screen from the slits is 1 m. If light of wavelength  $6000 \text{ \AA}$  is used, find the distance between the second dark fringe and the fourth bright fringe.

## Theory with Illustrations

Elaborative and simple theory helps the students to understand the illustrations supporting the theory. Please note that theory and problem solving techniques are based on simple learning program IF  $\rightarrow$  THEN  $\rightarrow$  ELSE. I would suggest you not to attempt the illustrations without going through the theory of that section.

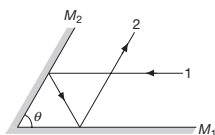


### Test Your Concepts-I

#### Based on Reflection at Plane Surfaces

(Solutions on page H.3)

1. A ray of light travelling in the direction  $\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})$  is incident on a plane mirror. After reflection, it travels along the direction  $\frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$ . Find the angle of incidence.
2. A ray of light travels from point A to a point B after



### Test Your Concepts-I

#### Based on

1. In a Young's Double Slit Experiment carried out in a liquid of refractive index  $\mu = 1.3$ , a thin film of thickness  $t$  is formed in front of the lower slit as shown in the figure. If a maxima of third order is formed at the origin O, find the
  - (a) thickness of the air film.
  - (b) positions of the fourth maxima.
 The wavelength of light in air is  $\lambda_0 = 0.78 \mu\text{m}$  and  $\frac{D}{d} = 1000$ .

### Test Your Concepts-II

#### Based on

1. A slit of width 0.025 m is placed in front of a lens of focal length 50 cm. The slit is illuminated with light of wavelength 5900 Å. Calculate the distance between the centre and first dark band of diffraction pattern obtained on a screen placed at the focal plane of the lens.
2. Two spectral lines of sodium  $D_1$  and  $D_2$  have wavelengths of approximately 5890 Å and 5896 Å. A sodium lamp sends incident plane wave onto a slit of width 2 micrometre. A screen is located 2 m from the slit. Find the spacing between the first

### Test Your Concepts

These topic based exercise sets are based on simple, single concept classification technique. These are meant for students practice after they study a particular topic and want to practice more on that topic learnt. Finally, in case of any difficulty they can refer to the hints and solutions to these exercise sets given at the end of the book.

### Problem Solving Techniques

These techniques ensure that students become capable enough to solve a variety of problems in an easy and quick manner.

#### Problem Solving Technique(s)

- (a) **Basic Problems in Optics:** Most of the problems asked in optics expect us to find the position and nature of the final image formed by certain optical systems for a given object. The optical system may be just a mirror, or a lens or a combination of several reflecting and refracting surfaces.
- (b) **Basic Strategy for Solving the Problems:** To handle these kinds of problems, first of all, we identify the sequence in which the reflection and refraction are taking place. The several events of reflection or refraction can be named as Event 1, Event 2 and so on following the sequence in which they occur.

#### Problem Solving Technique(s)

- (a) The deviation is maximum for normal incidence i.e., when  $i = 0$  then,  $\delta = \delta_{\text{max}} = 180^\circ$ .
- (b) The deviation is minimum for grazing incidence i.e., when  $i \rightarrow \frac{\pi}{2}$ , then  $\delta = \delta_{\text{min}} = 0^\circ$ .
- (c) While dealing with the case of multiple reflections suffered by a ray, the net deviation suffered by the incident ray is the algebraic sum of deviation due to each single reflection. So,

$$\delta_{\text{total}} = \sum \delta_{\text{individual reflection}}$$

DO NOT FORGET TO TAKE INTO ACCOUNT THE SENSE OF ROTATION WHILE SUMMING UP THE

#### Problem Solving Technique(s)

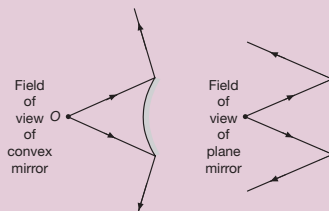
- (a) Interference occurs due to Law of Conservation of Energy. Actually, redistribution of energy takes place.
- (b) If  $w_1$  and  $w_2$  are the widths of the slits and  $I_1$  and  $I_2$  is the intensity of light (with respective amplitudes  $a_1$  and  $a_2$ ) passing through slits, then

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{w_1}{w_2}$$



**W Conceptual Note(s)**

It has been observed that a convex mirror gives a wider field of view than a plane mirror. Therefore, convex mirrors are used as rear view mirrors in vehicles. Though they make the estimation of distance more difficult but still they are preferred because a large movement of the object vehicle there is a small movement of the image.



**W Conceptual Note(s)**

To calculate the number of maximas or minimas that can be obtained on the screen, we use the fact that value of  $\sin\theta$  (or  $\cos\theta$ ) can never be greater than 1. For example in the first case when the slits are vertical.

$$\sin\theta = \frac{n\lambda}{d} \quad \{\text{for maximum intensity}\}$$

Since,  $\sin\theta \neq 1$

$$\Rightarrow \frac{n\lambda}{d} \neq 1$$

$$\Rightarrow n \neq \frac{d}{\lambda}$$

**Conceptual Notes**

The Conceptual Notes, Remarks, Words of Advice, Misconception Removals provide warnings to the students about common errors and help them avoid falling for conceptual pitfalls.

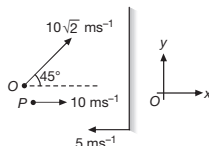
**Chapter End Solved Problems**

These are based on multiple concept usage in a single problem approach so as to expose a student's brain to the ultimate throttle required to take the JEE examination.

**SOLVED PROBLEMS**

**PROBLEM 1**

A plane mirror is moving with a uniform speed of  $5 \text{ ms}^{-1}$  along negative  $x$ -direction and an observer  $P$  is moving with a velocity of  $10 \text{ ms}^{-1}$  along  $+x$  direction. Calculate the velocity of image of an object  $O$ , moving with a velocity of  $10\sqrt{2} \text{ ms}^{-1}$  as shown in the figure, as observed by the observer. Also find its magnitude and direction.



Further, parallel to the mirror, i.e., along  $y$ -axis, we have

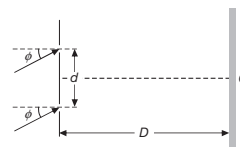
$$(\vec{v}_I)_y = (\vec{v}_O)_y = 10\hat{j}$$

Since  $\vec{v}_I = (\vec{v}_I)_x + (\vec{v}_I)_y$

So, absolute velocity of the image is

**PROBLEM 4**

Light of wavelength  $\lambda = 500 \text{ nm}$  falls on two narrow slits placed a distance  $d = 50 \times 10^{-4} \text{ cm}$  apart, at an angle  $\phi = 30^\circ$  relative to the slits shown in figure. ON the lower slit a transparent slab of thickness  $0.1 \text{ nm}$  and refractive index  $\frac{3}{2}$  is placed. The interference pattern is observed on a screen at a distance  $D = 2 \text{ m}$  from the slits.



**PROBLEM 2**

An interference pattern is observed due to two coherent sources  $S_1$  placed at origin and  $S_2$  placed at  $(0, 3\lambda, 0)$ , where  $\lambda$  is the wavelength of the sources. A detector  $D$  is moved along the positive  $x$ -axis. Find the coordinates on the  $x$ -axis (excluding

## PRACTICE EXERCISES

## SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions which ONLY ONE is correct.

1. A transparent hemisphere has a radius of 8 cm and an index of refraction of 1.5. A point source  $O$  is placed on the axis half the distance from the flat surface and the spherical surface.

## LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions followed by questions. Each question has four choices (A), (B), (C) and (D). In this section, competitiveness there may be a few questions that may have

## Comprehension I

A ray of light is incident at  $45^\circ$  on the face  $AB$  of an

## MULTIPLE CORRECT CHOICE TYPE QUESTION

This section contains Multiple Correct Choice Type Questions which ONE OR MORE is/are correct.

1. The  $x$ - $y$  plane is the boundary between two transparent media. Medium 1 with  $z \geq 0$  has a refractive index  $n_1$  and medium 2 with  $z < 0$  has a refractive index  $n_2$ . A ray of light is incident on the plane of separation from medium 1 given by the vector  $\vec{r}$ . The angle of reflection is  $\theta$ .

## MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in COLUMN-I are labelled A, B, C and D, while the statements in COLUMN-II are labelled p, q, r, s (and t). Any given statement in COLUMN-I can have correct matching with ONE OR MORE statement(s) in COLUMN-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

## REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having correct. Each question contains STATEMENT 1 and STATEMENT 2. The bubbles to be darkened are as follows:

**Bubble (A)** If both statements are TRUE and STATEMENT 2 is the correct reason for STATEMENT 1.

**Bubble (B)** If both statements are TRUE but STATEMENT 2 is not the correct reason for STATEMENT 1.

**Bubble (C)** If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.

**Bubble (D)** If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** A parallel beam of light traveling in air can be displaced laterally by a parallel transparent slab by distance more than the thickness of the plate.  
**Statement-2:** The lateral displacement of light traveling in air increases with rise in value of refractive index of the slab.

## INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value given in the question(s).

1. Two plane mirrors  $A$  and  $B$  are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle  $30^\circ$  at a point just inside one end of  $A$ . The plane of incidence coincides with the plane of the figure. Find the maximum number of times the ray undergoes reflections (including the first one) before it emerges out.

## Practice Exercise

Inclusion of all types of questions asked in JEE Advanced in adequate numbers helps you with enough practice

## Archive JEE Main and Advanced

From this fully updated section, students get to know the actual pattern of the problems asked in the past examinations.

## ARCHIVE: JEE MAIN

1. [Online April 2019] In figure, the optical fiber is  $l = 2$  m long and has a diameter of 1 cm. A light ray is incident at one end of the fiber at an angle of reflection  $40^\circ$ .

## ARCHIVE: JEE ADVANCED

## Single Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which only one is correct.)

1. [JEE (Advanced) 2019] A small convex lens of refractive index  $n$  is placed on the surface of a liquid of refractive index  $n_1$  such that the optical axis of the lens is perpendicular to the surface of the liquid.

## Matrix Match/Column Match Type Questions

Each question in this section contains statements given in two columns, which have to be matched. The statements in COLUMN-I are labelled A, B, C and D, while the statements in COLUMN-II are labelled p, q, r, s (and t).

## Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

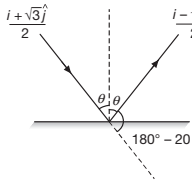
1. [JEE (Advanced) 2019]

A planar structure of length  $L$  and width  $W$  is made of two different optical media of refractive indices  $n_1 = 1.5$  and  $n_2 = 1.44$  as shown in figure. If  $L \gg W$ , a ray entering from end  $AB$  will emerge from end  $CD$  only if the total internal reflection condition is met inside the structure. For  $L = 9.6$  m, if the incident angle  $\theta$  is varied, the maximum time taken by a ray

**CHAPTER 1: RAY OPTICS**

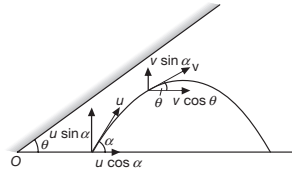
**Test Your Concepts-I  
(Based on Reflection at Plane Surfaces)**

1. The angle between the incident ray and the reflected ray is  $180 - 2\theta$ , so, we have



$\Rightarrow x = 4$  cm  
So, point of incidence of light from A should be at 4 cm from D on mirror.

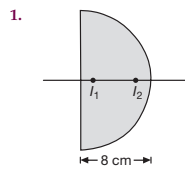
3. The image will be momentarily at rest when the particle moves parallel to the mirror. Let at the time  $t$  the particle has a velocity  $v$  parallel to the mirror.



**Hints and Explanations**

Exhaustive solutions with shortcuts (where ever needed), help students enhance their problem-solving skills.

**Single Correct Choice Type Questions**



Distance of image from the plane surface is

$$x_1 = \frac{4}{1.6} = 2.5 \text{ cm} \quad \left\{ \because d_{\text{app}} = \frac{d_{\text{actual}}}{\mu} \right.$$

For the curved surface, we have

$$\frac{1.6}{4} + \frac{1}{x_2} = \frac{1 - 1.6}{-8}$$

$$\Rightarrow x_2 \approx -3 \text{ cm}$$

The minus sign means the image is on the side where the object lies.

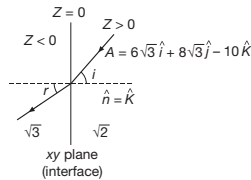
**Multiple Correct Choice Type Questions**

1. The normal to the interface is along  $\hat{k}$ .

$$\therefore \cos i = \frac{\vec{A} \cdot (-\hat{k})}{|\vec{A}| |\hat{k}|} = \frac{+10}{20} = +\frac{1}{2}$$

$$\Rightarrow i = 60^\circ$$

( $i$  is the angle which incident ray makes with  $-Z$ -axis)



But according to Snell's Law

$$\sqrt{2} \sin 60 = \sqrt{3} \sin r$$

# PREFACE

In the past few years, the IIT-JEE has evolved as an examination designed to check a candidate's true scientific skills. The examination pattern needs one to see those little details which others fail to see. These details tell us how much in-depth we should know to explain a concept in the right direction. Keeping the present-day scenario in mind, this series is written for students, to allow them not only to learn the tools but also to see why they work so nicely in explaining the beauty of ideas behind the subject. The central goal of this series is to help the students develop a thorough understanding of Physics as a subject. This series stresses on building a rock-solid technical knowledge based on firm foundation of the fundamental principles followed by a large collection of formulae. The primary philosophy of this book is to guide the aspirants towards detailed groundwork for strong conceptual understanding and development of problem-solving skills like mature and experienced physicists.

This updated Third Edition of the book will help the aspirants prepare for both Advanced and Main levels of JEE conducted for IITs and other elite engineering institutions in India. This book will also be equally useful for the students preparing for Physics Olympiads.

This book is enriched with detailed exhaustive theory that introduces the concepts of Physics in a clear, concise, thorough and easy-to-understand language. A large collection of relevant problems is provided in eight major categories (including updated archive for JEE Advanced and JEE Main), for which the solutions are demonstrated in a logical and stepwise manner.

We have carefully divided the series into seven parts to make the learning of different topics seamless for the students. These parts are

- Mechanics – I
- Mechanics – II
- Waves and Thermodynamics
- Electrostatics and Current Electricity
- Magnetic Effects of Current and Electromagnetic Induction
- Optics
- Modern Physics

Finally, I would like to thank all my teacher friends who had been a guiding source of light throughout the entire journey of writing this book.

To conclude, I apologise in advance for the errors (if any) that may have inadvertently crept in the text. I would be grateful to the readers who bring errors of any kind to my attention. I truly welcome all comments, critiques and suggestions. I hope this book will nourish you with the concepts involved so that you get a great rank at JEE.

PRAYING TO GOD FOR YOUR SUCCESS AT JEE. GOD BLESS YOU!

**Rahul Sardana**

# ABOUT THE AUTHOR

**Rahul Sardana**, is a Physics instructor and mentor having rich and vast experience of about 20 years in the field of teaching Physics to JEE Advanced, JEE Main and NEET aspirants. Along with teaching, authoring books for engineering and medical aspirants has been his passion. He authored his first book '*MCQs in Physics*' in 2002 and since then he has authored many books exclusively for JEE Advanced, JEE Main and NEET examinations.

He is also a motivational speaker having skills to motivate students and ignite the spark in them for achieving success in all colours of life. Throughout this journey, by the Grace of God, under his guidance and mentorship, many of his students have become successful engineers and doctors.

# Ray Optics

## Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

- |  |  |
|--|--|
| (a) Reflection for plane and curved surfaces (i.e. for plane and curved mirrors) | (d) Lens   |
| (b) Refraction for plane surfaces (i.e. for glass slab and prism)                | (e) Lens Makers Formula                          |
| (c) Refraction for curved surfaces   | (f) Human eye                                    |
|  | (g) Defects in human eye and optical instruments |

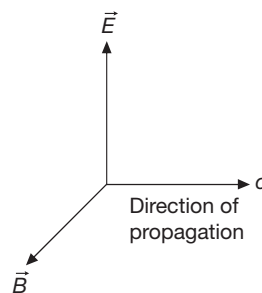
All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

## REFLECTION AT PLANE AND CURVED SURFACES

### NATURE OF LIGHT: AN INTRODUCTION

Light is a form of energy that makes object visible to our eyes or light is the form of energy that produces in us the sensation of sight. In Seventeenth century **Newton** and **Descartes** believed that light consisted of a stream of particles, called corpuscles. **Huygens** proposed wave theory of light and proposed that light is a disturbance in a medium called **Ether**. This theory could explain the phenomena of interference, diffraction, etc. Thomas Young, through his double slit experiment, measured the wavelength of light.

**Maxwell** suggested the electromagnetic theory of light. According to this theory, light consists of electric and magnetic fields, in mutually perpendicular directions, and both are perpendicular to the direction of propagation. Heinrich Hertz produced in the laboratory the electromagnetic waves of short wavelengths. He showed that these electromagnetic waves possessed all the properties of light waves.



Light travels in vacuum with a velocity given by

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ ms}^{-1}$$

where  $\mu_0$  and  $\epsilon_0$  are the permeability and permittivity of free space (vacuum).

The magnitudes of electric and magnetic fields are related to the velocity of light by the relation

$$\frac{E}{B} = c$$

In 1905, **Albert Einstein** revived the old corpuscular theory using **Plank's Quantum Hypothesis** and through his photoelectric effect experiment showed that light consists of discrete energy packets, called **photons**. The energy of each photon is

$$E = hf = \frac{hc}{\lambda}$$

So, in view of these developments, light must be regarded to have a dual nature i.e., it exhibits the characteristics of a particle in some situations and that of a wave in other situations. So the question “**Is light a particle or a wave?**” is purely inappropriate to be asked. At present, it is believed that light has **dual nature**, i.e., it has both the characters, wave-like and particle-like.

## OPTICS: AN INTRODUCTION

Optics is the study of the properties of light, its propagation through different media and its effects. In most of the situations, the light encounters objects of size much larger than its wavelength. We can assume that light travels in straight lines called **rays**, disregarding its wave nature. This allows us to formulate the rules of optics in the language of **geometry**, as rays of light do not disturb each other on intersection. Such study is called **geometrical (or ray) optics**. It includes the working of mirrors, lenses, prisms, etc.

When light passes through very narrow slits, or when it passes around very small objects, we have to consider the wave nature of light. This study is called **wave (or physical) optics**.

## DOMAINS OF OPTICS

The study of light can be categorized into three broad domains.

- (a) Geometrical Optics (Ray Optics)
- (b) Physical Optics (Wave Optics)
- (c) Quantum Optics

Please note that these domains are not strictly disjoint as the transitions between them are continuous and not sharp. However for convenience we consider them as distinct. These domains are distinguished as follows.

## Geometrical Optics (Ray Optics)

This branch involves the study of propagation of light based on the assumption that light travels in fixed straight line as it passes through a uniform medium and its direction is changed when met by a surface of a different medium or if the optical properties of the medium are non uniform either in time or in space. The ray approximation is valid for the wavelength  $\lambda$  very small compared to the size of the obstacle ( $d$ ) or the size of the opening through which the ray passes. This approximation  $\lambda \ll d$  proves to be very good for the study of mirrors, lenses, prisms and associated optical instruments such as microscope, telescope, cameras etc.

## Physical Optics (Wave Optics)

This branch involves the study of propagation of light in the form of a wave and it deals with the phenomenon of interference, diffraction, polarization etc. This nature of light has to be taken when the light passes through very narrow slits or when it goes past very small objects. So this branch works effectively when  $\lambda \gg d$ .

## Quantum Optics

This branch involves the study of propagation of light as a stream of particles called as **Photons**. This concept of light behaving as particles called photons is of utmost importance while studying the origin of spectra, photoelectric effect, concept of radiation pressure, Compton effect etc.

## FUNDAMENTAL LAWS OF GEOMETRICAL OPTICS

To a first approximation, we can consider the propagation of light disregarding its wave nature and assuming that light propagates in straight lines called **rays**. This allows us to formulate the laws of optics in the language of **geometry**. Thus, the branch of optics where the wave nature of light is neglected is called **geometrical (or ray) optics**.

**Geometrical optics is based on five fundamental laws.**

- 1. Law of Rectilinear Propagation of Light.** It states that light propagates in straight lines in homogenous media.
- 2. Law of Independence of Light Rays.** It states that rays do not disturb each other upon intersection.
- 3. The Law of Reversibility of Light.** According to this law, if a ray of light, after suffering a number of reflections and refractions, has its path reversed at any instant, then the ray retraces its path back to the source.
- 4. The Laws of Reflection.** The Laws of Reflection govern the bouncing back of the incident ray after striking a surface to the medium from which it was coming.
- 5. The Laws of Refraction (discussed later).** The Laws of Refraction govern the bending of light when the light goes from one medium to the other (rarer to denser or denser to rarer) medium.

## BASIC TERMS AND DEFINITIONS

### Source

A body which emits light is called **source**. The source can be a point one or an extended one. A source is of two types.

- (a) Self luminous:** The source which possess light of its own.  
**EXAMPLE:** sun, electric arc, candle etc.
- (b) Non-luminous:** It is a source of light which does not possess light of its own but acts as source of light by reflecting the light received by it.  
**EXAMPLE:** moon, objects around us, book etc.

### Remark(s)

Sources are also classified as isotropic and non-isotropic. Isotropic sources give out light uniformly in all directions whereas non-isotropic sources do not give out light uniformly in all directions.

## RAY

The straight line path along which the light travels between two points in a homogeneous medium or in a pair of media is called a Ray. It is represented

by an arrow head on a straight line, the arrow head represents the direction of propagation of light. A ray of light will always follow a path along which the time taken is the minimum.



### Remark(s)

A single ray cannot be isolated from a source of light.

## MEDIUM

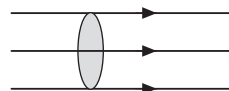
Substance through which light propagates or tends to propagate is called a **medium**. It is of following three types.

- (a) Transparent:** It is a medium through which light can be propagated easily.  
**EXAMPLE:** glass, water etc.
- (b) Translucent:** It is a medium through which light is propagated partially.  
**EXAMPLE:** oil paper, ground glass etc.
- (c) Opaque:** It is a medium through which light cannot be propagated.  
**EXAMPLE:** wood, iron etc.

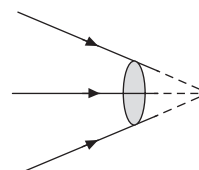
## BEAM

A bundle or bunch of rays is called a **beam**. It is of following three types.

- (a) Parallel beam:** It is a beam in which all the rays constituting the beam move parallel to each other and diameter of beam remains same. A very narrow beam is called a Pencil of Light.

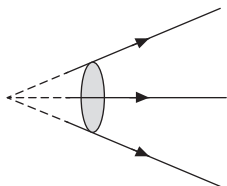


- (b) Convergent beam:** In this case diameter of beam decreases in the direction of ray.





- (c) **Divergent beam:** It is a beam in which all the rays meet at a point when produced backward and the diameter of beam goes on increasing as the rays proceed forward.

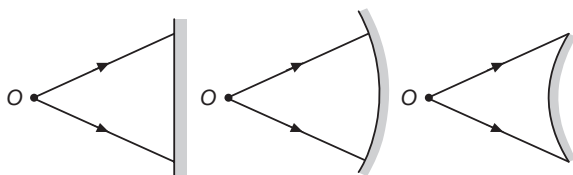


## OBJECT(S)

The object for a mirror can be real or virtual. Generally we can define an object as the point where the incident rays intersect (real object) or appear to intersect (virtual object).

### Real Object(s)

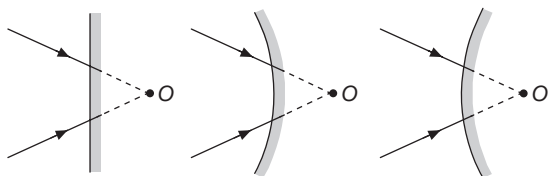
If the rays from a point on an object actually diverge from it and fall on the mirror, the object is said to be real.



In simple language the incident rays are diverging and the point of divergence is the position of the real object. The following diagrams support the arguments given.

### Virtual Object(s)

If the rays incident on the mirror appear to converge to a point, then this point is said to be virtual point object for the mirror.



In simple language the incident rays are converging and the point of convergence is the position of the virtual object. The following diagrams support the arguments given.



## Conceptual Note(s)

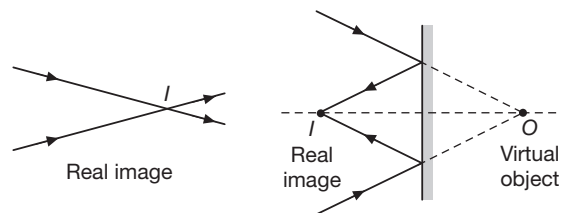
Virtual object cannot be seen by human eye, because for an object or an image to be seen by the eyes, the rays received by the eyes must be diverging.

## IMAGE(S)

An **optical image** is a point where reflected or refracted rays of light either intersect or appear to intersect. Thus, the image of an infinite object is actually an assembly of the image points corresponding to various parts or the points of the object. The images formed can again be real or virtual.

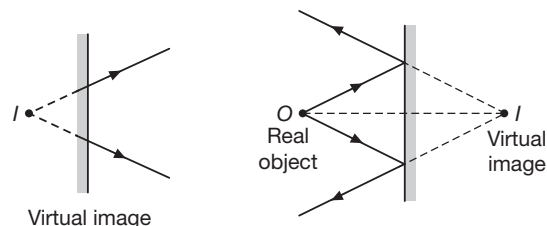
### Real Image(s)

If the rays after reflection or refraction actually converge (or meet) at a point then the image is said to be **real** and it can be obtained on a screen.



### Virtual Image(s)

However, if the rays do not actually converge but appear to diverge from a point (or appear to meet at a point), then the image so formed is said to be **virtual image**. A virtual image cannot be obtained on a screen.



## Conceptual Note(s)

- (a) The real images can be obtained on a suitably placed screen, but virtual images cannot be obtained on a screen.
- (b) Human eye cannot distinguish between the real image and the virtual image because in both the cases the rays are diverging.

## REFLECTION OF LIGHT

When light strikes the surface on an object, some part of the light or the complete light is sent back into the same medium. This phenomenon is called as reflection. The surface, which reflects light, is called mirror. A mirror could be plane or curved.

## Conceptual Note(s)

In reflection, the frequency, speed and wavelength remain unchanged, but a phase change may occur depending on the nature of reflecting surface.

The reflection from a denser medium causes an addition phase change of  $\pi$  or a path change of  $\frac{\lambda}{2}$  (by Stoke's Law) while reflection from rarer medium does not cause any phase change.

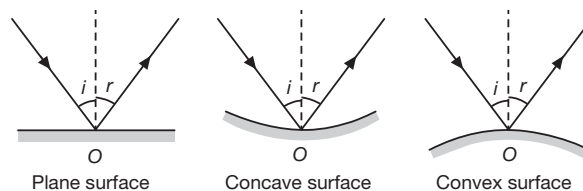
**Diffused** (irregular) reflection takes place from a rough surface whereas **Specular** (regular) reflection takes place from an extraordinarily smooth surface. However, the Laws of Reflection are applicable for both kinds of surfaces.

## LAWS OF REFLECTION

- (a) The incident-ray, the reflected-ray and the normal to the reflecting surface at the point of incidence, all lie in the same plane.
- (b) The angle of reflection is equal to the angle of incidence ( $i = r$ ).

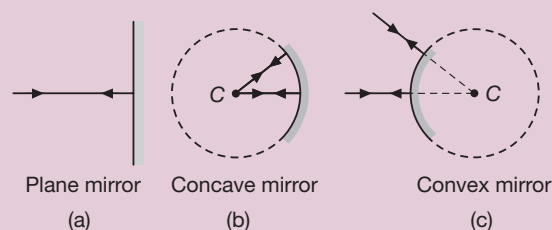
The angle of incidence  $i$  is the angle made by the incident ray with the normal.

The angle of reflection  $r$  is the angle made by the reflected ray with the normal.



## SPECIAL CASES

- (a) If  $i = 0$ , then  $r = 0$ . It means a ray incident normally on a boundary, after reflection it retraces its path.



- (b) The angle made by the incident ray with the plane reflecting surface is called glancing angle. Thus, the glancing angle  $= 90^\circ - i$ .
- (c) For grazing incidence, the incident ray grazes the reflecting surface, so  $i \rightarrow \frac{\pi}{2}$  and hence  $r \rightarrow \frac{\pi}{2}$  as shown in the figure.



## FERMAT'S PRINCIPLE OF LEAST TIME

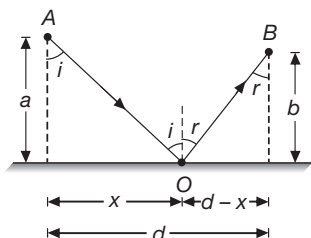
According to this theorem, the path of a ray of light between any two points is the path along which the time taken is the minimum. This principle is sometimes taken as the definition of a ray of light.

To understand this theorem, let us consider two points  $A$  and  $B$  in the same medium. Since, we know that between these two points light travels in a straight line, so the time taken by the light to go from  $A$  to  $B$  must logically be the minimum.



## LAWS OF REFLECTION USING FERMAT'S THEOREM

Consider a plane mirror on which light is incident as shown.



Let the incident light start from  $A$ , hit the mirror at  $O$  and get reflected to point  $B$ . Let the points  $A$  and  $B$  be at perpendicular distances  $a$  and  $b$  from the mirror and let  $A$  and  $B$  have a separation  $d$  between them as shown in figure. The time taken by the light to go from  $A$  to  $O$  to  $B$  is given by

$$t = t_{A \rightarrow O} + t_{O \rightarrow B}$$

$$\Rightarrow t = \frac{AO}{c} + \frac{OB}{c}$$

$$\Rightarrow t = \frac{1}{c} \left( \sqrt{a^2 + x^2} + \sqrt{b^2 + (d-x)^2} \right)$$

Now, according to Fermat's Principle,  $t$  is MINIMUM, so

$$\frac{dt}{dx} = 0$$

$$\Rightarrow \frac{d}{dx} \left( \sqrt{a^2 + x^2} \right) + \frac{d}{dx} \left( \sqrt{b^2 + (d-x)^2} \right) = 0$$

$$\Rightarrow \frac{1}{2} \left( \frac{2x}{\sqrt{a^2 + x^2}} \right) + \frac{1}{2} \left( \frac{2(d-x)(-1)}{\sqrt{b^2 + (d-x)^2}} \right) = 0$$

$$\Rightarrow \frac{x}{\sqrt{a^2 + x^2}} = \frac{(d-x)}{\sqrt{b^2 + (d-x)^2}}$$

From the figure, we observe that

$$\frac{x}{\sqrt{a^2 + x^2}} = \sin i \quad \text{and} \quad \frac{(d-x)}{\sqrt{b^2 + (d-x)^2}} = \sin r$$

$$\Rightarrow \sin i = \sin r$$

$$\Rightarrow i = r$$

**{The Law of Reflection}**

## Problem Solving Technique(s)

**(a) Basic Problems in Optics:** Most of the problems asked in optics expect us to find the position and nature of the final image formed by certain optical systems for a given object. The optical system may be just a mirror, or a lens or a combination of several reflecting and refracting surfaces.

**(b) Basic Strategy for Solving the Problems:** To handle these kinds of problems, first of all, we identify the sequence in which the reflection and refraction are taking place. The several events of reflection or refraction can be named as Event 1, Event 2 and so on following the sequence in which they occur.

Now, the image of Event 1 would be object for Event 2, image of Event 2 will be object of Event 3 and so on. This way one can proceed to find the final image.

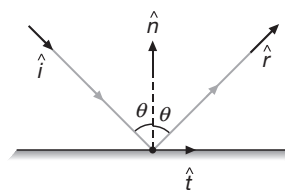
## VECTOR FORM OF LAWS OF REFLECTION

Laws of reflection can be redefined with the help of vector algebra by considering unit vectors in the direction of incident rays, reflected rays and normal to the boundary.

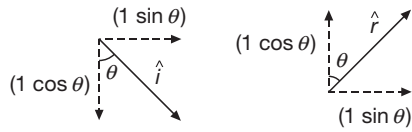
The reflection of a light ray incident on a plane surface is shown in figure. If  $\hat{i}$ ,  $\hat{r}$  and  $\hat{n}$  are unit vectors along the direction of incident ray, reflected ray and normal to the surface as shown, then first we can write components of  $\hat{i}$  and  $\hat{r}$  in terms of the unit vectors along the normal and along the surface i.e. tangential to surface. Let  $\hat{t}$  be a unit vector tangential to the surface, so we have

$$\hat{i} = (\sin \theta) \hat{t} - (\cos \theta) \hat{n} \quad \dots(1)$$

$$\hat{r} = (\sin \theta) \hat{t} + (\cos \theta) \hat{n} \quad \dots(2)$$



$$|\hat{i}| = |\hat{r}| = |\hat{n}| = |\hat{t}| = 1$$



Subtracting equation (1) from (2), we get

$$\hat{r} = \hat{i} + (2 \cos \theta) \hat{n} \quad \dots(3)$$

Also, we know that

$$\hat{i} \cdot \hat{n} = |\hat{i}| |\hat{n}| \cos(180 - \theta) = -\cos \theta \quad \dots(4)$$

Substituting (4) in (3), we get

$$\hat{r} = \hat{i} - 2(\hat{i} \cdot \hat{n}) \hat{n}$$

This equation gives us the Laws of Reflection in vector form.

### ILLUSTRATION 1

A ray of light is incident on a plane mirror along a vector  $\hat{i} + \hat{j} - \hat{k}$ . The normal at the point of incidence is along  $\hat{i} + \hat{j}$ . Find a unit vector along the reflected ray.

### SOLUTION

Reflection of a ray of light is just like an elastic collision of a ball with a horizontal ground. The component of the incident ray along the inside normal gets reversed while the component perpendicular to it remains unchanged. So, the component of incident ray vector  $\vec{A} = \hat{i} + \hat{j} - \hat{k}$  parallel to normal, i.e.,  $\hat{i} + \hat{j}$  gets reversed while perpendicular to it, i.e.,  $-\hat{k}$  remains unchanged. So, the reflected ray is written as,

$$\vec{R} = -\hat{i} - \hat{j} - \hat{k}$$

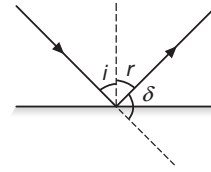
A unit vector along the reflected ray will be,

$$\hat{r} = \frac{\vec{R}}{R} = \frac{-\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$$

$$\Rightarrow \hat{r} = -\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$$

### ANGLE OF DEVIATION ( $\delta$ )

Deviation ( $\delta$ ) is defined as the angle between the initial direction of the incident ray and the final direction of the reflected ray or the emergent ray.

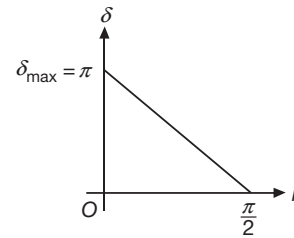


Deviation produced in Reflection is  $\delta = 180^\circ - (i + r)$

Since  $r = i$

$$\Rightarrow \delta = 180^\circ - 2i$$

The variation of deviation ( $\delta$ ) with the angle of incidence ( $i$ ) is shown in figure.



### Problem Solving Technique(s)

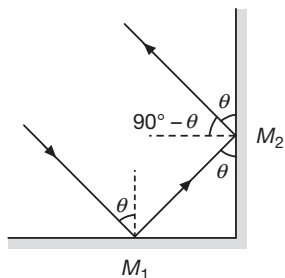
- The deviation is maximum for normal incidence i.e., when  $i = 0$  then,  $\delta = \delta_{\max} = 180^\circ$ .
- The deviation is minimum for grazing incidence i.e., when  $i \rightarrow \frac{\pi}{2}$ , then  $\delta = \delta_{\min} = 0^\circ$ .
- While dealing with the case of multiple reflections suffered by a ray, the net deviation suffered by the incident ray is the algebraic sum of deviation due to each single reflection. So,

$$\delta_{\text{total}} = \sum \delta_{\text{individual reflection}}$$

DO NOT FORGET TO TAKE INTO ACCOUNT THE SENSE OF ROTATION WHILE SUMMING UP THE DEVIATIONS DUE TO SINGLE REFLECTION.

### TWO IDENTICAL PERPENDICULAR PLANE MIRRORS

If two plane mirrors are inclined to each other at  $90^\circ$ , the emergent ray is always antiparallel to the incident ray if it suffers one reflection from each (as shown in figure) whatever be the angle of incidence.



### Conceptual Note(s)

The same is found to hold good for three plane mirrors arranged mutually perpendicular to each other thus forming the corner of a cube such that the light incident on this arrangement suffers one reflection from each of the mirrors so as to emerge out anti-parallel to the incident light. This arrangement of three mutually perpendicular plane mirrors forming the corner of a cube is called the CORNER REFLECTOR.

#### ILLUSTRATION 2

Two plane mirrors are inclined to each other at an angle  $\theta$ . A ray of light is reflected first at one mirror and then at the other. Find the total deviation suffered by the ray.

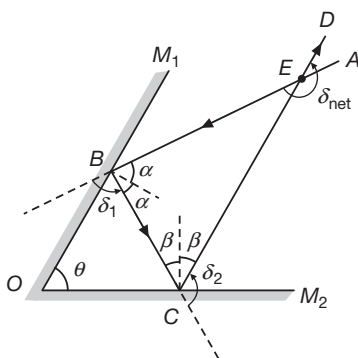
#### SOLUTION

$\alpha$  be the angle of incidence for mirror  $M_1$

$\beta$  be the angle of incidence for mirror  $M_2$

$\delta_1$  be the deviation due to mirror  $M_1$  and

$\delta_2$  be the deviation due to mirror  $M_2$



From figure, we observe

$$\delta_1 = \pi - 2\alpha, \quad \delta_2 = \pi - 2\beta$$

Also ray is rotated in same sense i.e., anticlockwise, so

$$\delta_{\text{net}} = \delta = \text{Total deviation} = \delta_1 + \delta_2$$

$$\Rightarrow \delta = 2\pi - 2(\alpha + \beta)$$

Now in  $\triangle OBC$ ,  $\angle OBC + \angle BCO + \angle COB = 180^\circ$

$$\Rightarrow (90^\circ - \alpha) + (90^\circ - \beta) + \theta = 180^\circ$$

$$\Rightarrow \alpha + \beta = \theta$$

$$\Rightarrow \delta = 2\pi - 2\theta = 360^\circ - 2\theta$$

**Alternative Method:**

$$\delta = \angle BEC + \angle CEA + \angle AED$$

Now,  $\angle BEC = \angle AED$  (vertically opposite angle)

$$\Rightarrow \angle BEC = 180^\circ - 2(\alpha + \beta)$$

$$\Rightarrow \angle BEC = 180^\circ - 2\theta \quad \{\because \theta = \alpha + \beta\}$$

Also,  $\angle CEA = 2\alpha + 2\beta \Rightarrow \angle CEA = 2(\alpha + \beta) = 2\theta$

$$\Rightarrow \delta = (180^\circ - 2\theta) + 2\theta + (180^\circ - 2\theta)$$

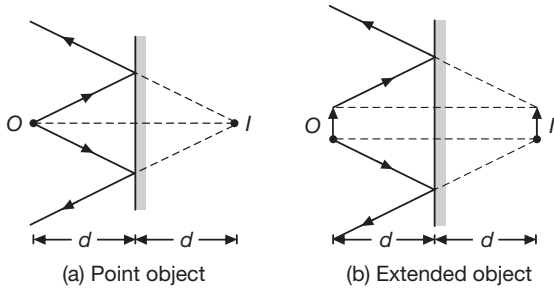
$$\Rightarrow \delta = 360^\circ - 2\theta$$

### Conceptual Note(s)

If two mirrors are inclined at  $\angle \theta$  then the ray incident on any one mirror will suffer a total deviation  $\delta = 2\pi - 2\theta$  after suffering reflection from both of the mirrors.

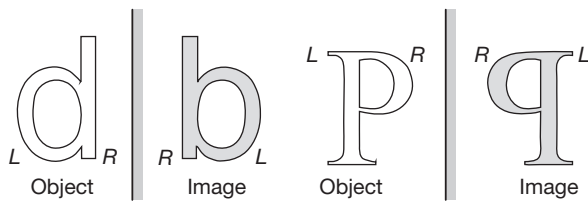
### REFLECTION FROM A PLANE SURFACE OR PLANE MIRROR

When a real object is placed in front of a plane mirror, the image is always erect, virtual and of same size as the object. It is at same distance behind the mirror as the object is in front of it.

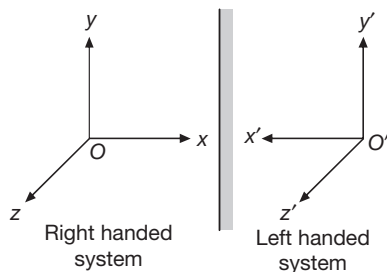


### LATERAL INVERSION

The image formed by a plane mirror suffers **lateral-inversion**. That is, in the image the left is turned to the right and vice-versa with respect to object. However, the plane mirror does not turn up and down, as shown in figure.

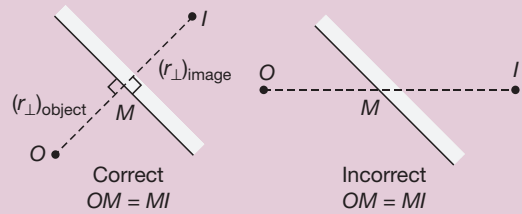


Actually, the plane mirror reverses forward and back in three-dimensions (and not left into right). If we keep a right-handed coordinate system in front of a plane mirror, only the z-axis is reversed. So, a plane mirror changes right-handed co-ordinate system (or screw) to left-handed.



### Problem Solving Technique(s)

(a) For finding the location of an image of a point object placed in front of a plane mirror, we must see the perpendicular distance of the object from the mirror.

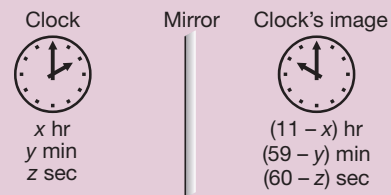


$$\text{i.e., } \left( \perp \text{ distance of } \begin{matrix} O \\ \text{from mirror} \end{matrix} \right) = \left( \perp \text{ distance of } \begin{matrix} I \\ \text{from mirror} \end{matrix} \right)$$

(b) When a wall clock is seen in a mirror then

$$\text{Image Time} = 11 \text{ hour } 60 \text{ minute} - \text{Actual Time}$$

$$\Rightarrow t_{\text{image}} = 11:60 - t_{\text{actual}}$$



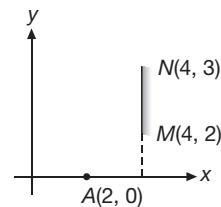
**Example 1:** If actual time in the clock is 5 : 25, then the image of the clock in the plane mirror will show a time given by

$$t_{\text{image}} = 11:60 - 5:25 = 6:35$$

**Example 2:** If actual time in the clock is 2 hr, 17 min, 25 sec then the image clock will show a time of 9 hr, 42 min, 35 sec.

### ILLUSTRATION 3

An object is lying at  $A(2, 0)$  and  $MN$  is a plane mirror, as shown. Find the region on  $Y$ -axis in which reflected rays are present.



### SOLUTION

The image of point  $A$ , in the mirror is at  $A'(6, 0)$ .