fifth edition

Precalculus

A Right Triangle Approach

Ratti
McWaters
Skrzypek

with

Bernards
Fresh

FIFTH EDITION

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A Right Triangle Approach

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Foreword

We're pleased to present the fifth edition of *Precalculus: A Right Triangle Approach*. Our experience in teaching this material has been exceptionally rewarding. Because students are accustomed to information being delivered by electronic media, the introduction of MyLab™ Math into our courses was, and remains, seamless. With the addition of Jessica Bernards and Wendy Fresh to the author team, the MyLab course has been given a fresh redesign that aligns with the Ratti philosophy. You will now find author created videos over every objective as well as author created assignments, quizzes, and exams. Additionally, we have included interactive figures in both the print and electronic version of the text that will allow students to get a hands on exploration of the topics.

Today's precalculus students and instructors face many challenges. Students arrive with various levels of comprehension from their previous courses. Instead of really learning the concepts presented, students often resort to memorization to pass the course. As a result, a course needs to establish a common starting point for students and engage them in becoming active learners, without sacrificing the solid mathematics essential for conceptual understanding. Instructors in this course must take on the task of providing students with an understanding of precalculus, preparing them for the next step, and ensuring that they find mathematics useful and interesting. Our efforts in this direction have been aided considerably by the many suggestions we have received from users of the previous editions of this text.

Mathematics owes it current identity to contributions from diverse cultures across the world and throughout the ages. In this text we provide references to significant improvements and achievements in mathematics and related areas from sources both ancient and modern. We place a strong emphasis on both concept development and real-life applications. Topics such as functions, graphing, the difference quotient, and limiting processes provide thorough preparation for the study of calculus and will improve students' comprehension of algebra. Just-in-time review throughout the text ensures that all students are brought to the same level before being introduced to new concepts. Numerous applications motivate students to apply the concepts and skills they learn in precalculus to other courses, including the physical and biological sciences, engineering, and economics, and to on-the-job and everyday problem solving. Students are given ample opportunities in this course to think about important mathematical ideas and to practice and apply algebraic skills.

Throughout the text, we emphasize why the material being covered is important and how it can be applied. By thoroughly developing mathematical concepts with clearly defined terminology, students see the "why" behind those concepts, paving the way for a deeper understanding, better retention, less reliance on rote memorization, and ultimately more success. The level of exposition was selected so that the material is accessible to students and provides them with an opportunity to grow.

It is our hope that once you have read through our text, you will see that we were able to fulfill the initial goals of writing for today's students and for you, the instructor.

Marcus McWaters **Lesław Skrzypek** **J. S. Ratti**

Jessica Bernards **Wendy Fresh**

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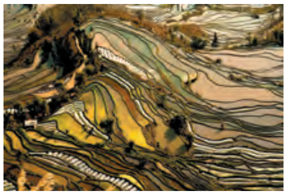

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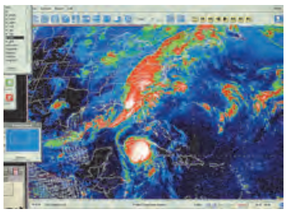

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Preface

Students begin precalculus classes with widely varying backgrounds. Some haven't taken a math course in several years and may need to spend time reviewing prerequisite topics, while others are ready to jump right into new and challenging material. In Chapter P and in some of the early sections of other chapters, we have provided review material in such a way that it can be used or omitted as appropriate for your course. In addition, students may follow several paths after completing a precalculus course. Many will continue their study of mathematics in courses such as finite mathematics, statistics, and calculus. For others, precalculus may be their last mathematics course.

Responding to the current and future needs of all these students was essential in creating this text. We introduce each exercise set with several concept and vocabulary exercises, consisting of fill-in-the-blank and true-false exercises. They are not computation-reliant, but rather test whether students have absorbed the basic concepts and vocabulary of the section. Exercises asking students to extrapolate information from a given graph now appear in much greater number and depth throughout the course. We continue to present our content in a systematic way that illustrates how to study and what to review. We believe that if students use this material well, they will succeed in this course. The changes in this edition result from the thoughtful feedback we have received from students and instructors who have used previous editions of the text. This feedback crucially enhances our own experiences, and we are extremely grateful to the many contributors whose insights are reflected in this new edition.

Key Content Changes

New! Authors. We would like to welcome two new co-authors to the author team, Jessica Bernards and Wendy Fresh! Both Jessica and Wendy are instructors at Portland Community College and have provided wonderful additions to the text and the accompanying MyLab Math course. Jessica and Wendy have had national recognition as instructors and have received several awards for excellence in teaching mathematics.

Revised! Getting Ready for the Next Section. This feature combines two features of previous editions, "Getting Ready for the Next Section" exercises and "Before Starting This Section, Review" objectives. The new structure lists Review Concepts and Review Skills for students to brush up on before beginning the next section. Both the Review Concepts and Review Skills contain section and page number references to make looking up these topics easy. The Review Concepts are meant to be broader topics that students should understand, while Review Skills give exercises from objectives that students will need in the upcoming section.

New! Key Ideas at a Glance. A new feature added to the text for this edition is a single page at the end of each chapter designed to highlight some key concepts for the chapter. In some chapters, this will serve as a comparison between two similar or parallel topics. In other chapters, this will sum up many of the ideas presented in the chapter. This page can serve as a reference to students to look back on when studying or doing exercises. There are also exercises to accompany this feature so that students may test their understanding of the ideas summarized there.

New and Revised! Exercises. We continue to improve the balance of exercises, providing a smooth transition from the less challenging to the more challenging exercises.

Overall, approximately 20% of the exercises have been updated, and more than 500 brand-new exercises have been added. These new exercises primarily consist of applications that connect with students' everyday experiences and enhance students' understanding of graphing.

Revised! Application Exercises. Every section opens with discussion of an application that relates to the topics introduced in that section. This edition continues the trend of pairing an example with this application, but we have also made an effort to include problems in the exercise sets that also tie to this application so that students have an opportunity to apply the mathematics to a real world problem. The section opening, example, and these exercises are easily identified by an accompanying icon .

New! Active Learning Exercises. In many sections throughout the text, exercise sets will end with an Active Learning exercise. This exercise is accompanied by an Interactive Figure powered by GeoGebra, which is accessed through a bit.ly link or by scanning the given QR code. Students will manipulate the figure to explore mathematics in a new way and will use the figure to answer the accompanying exercises.

New! Videos. Videos in the MyLab course have been completely re-made by the authors. Videos can be found at the section and objective level. The videos are available in the MyLab course within the Video & Resource Library, but can also be accessed directly from the text. QR codes can be found at the beginning of each section, and users can use their phone to scan the QR codes and watch the videos.

We have also created videos for select exercises in each section. The text has QR codes next to the beginning of each exercise set, and users can scan the code to find the videos for exercises in that section.

Revised! Diversity, Equity, and Inclusion. We conducted an external review of the text's content to determine how it could be improved to address issues related to diversity, equity, and inclusion. The results of that review informed the revision.

Chapter 1.

- Added a graphic to detail the process of factoring a trinomial.
- Added more Procedure by Action type examples which carefully breakdown factoring trinomials.
- Created a new section opener for section 1.5 highlighting female NASA engineers' current Mars Exploration Program

Chapter 2.

- Updated several examples and with newer application context.
- Expanded step by step transformation of functions procedure.
- Split a Procedure in Action example into two parts to give individual attention to finding a parallel line vs a perpendicular line to a given line.
- Included new Active Learning Exercises.

Chapter 3.

- Added more visual aid to the examples on graphing a polynomial, giving a more complete picture of the behavior of graphs of polynomials.
- Added more visual aid to the examples on graphing a rational function, giving a more complete picture of the behavior of graphs of rational functions.
- Included new Active Learning Exercises.

Chapter 4.

- Expanded on the basic properties of logarithms to give some derivations of these properties.
- Added color to the table of logarithmic functions to make it easier to see how changes in the function change the graph itself.
- Included new Active Learning Exercises.

Chapter 5.

- Updated graphs for sine and cosine functions to give additional detail around key points.
- Created new graphics to show the symmetries of the sine and cosine functions.
- Updated graphics showing stretch and compression of sine and cosine graphs.
- Updated graphics showing phase shifts and vertical shifts of sine and cosine graphs.
- Added additional graphics for the inverse sine, cosine, and tangent functions.
- Included new Active Learning Exercises.

Chapter 6.

- Added a graphic to demonstrate one of the Pythagorean identities.
- Added graphics to support trigonometric identities using the symmetries of the sine and cosine functions.
- Included new Active Learning Exercises.

Chapter 7

- Updated and added graphics and explanation for the ambiguous case of SSA triangles.
- Updated solution process for SSA triangles.
- Included new Active Learning Exercises.

Chapter 8.

- Wrote all new section opener discussions for sections 8.1 and 8.5.
- Added a new example in section 8.5 showing an application involving cell tower triangulation.
- Included new Active Learning Exercises.

Chapter 9.

- Added a link between the section opener discussion and an example in section 9.4.
- Included new Active Learning Exercises.

Chapter 10.

- Added many examples of conics seen and used in the real world.
- Included new Active Learning Exercises.

Chapter 11.

- Changed terms used in the sum of the first n terms of an arithmetic sequence so that the derivation of the formula is clearer.
- Added graphics to demonstrate the derivation of the sum of the first n terms of arithmetic and geometric sequences.
- Included new Active Learning Exercises.

Features

Chapter Opener. Each chapter opener includes a description of applications (one of them illustrated) relevant to the content of the chapter and the list of topics that will be covered. In one page, students see what they are going to learn and why they are learning it.

Getting Ready for the Next Section. Each section is immediately preceded by a set of concepts and skills that serve as a transition from one section to the next. These sets of problems provide a review of concepts and skills that will be used in the upcoming section.

Section Opener With Application. Each section opens with a list of clearly stated and numbered **Objectives** defined for the section. These objectives are then referenced again in the margin of the lesson at the point where the objective's topic is taught. An **Application** containing a motivating anecdote or an interesting problem then follows. An example later in the section relating to this application and identified by the same icon ( ) is then solved using the mathematics covered in the section. These applications utilize material from a variety of fields: the physical and biological sciences (including health sciences), economics, art and architecture, history, and more.

Examples and Practice Problems. **Examples** include a wide range of computational, conceptual, and modern applied problems carefully selected to build confidence, competency, and understanding. Every example has a title indicating its purpose and presents a detailed solution containing annotated steps. All examples are followed by a **Practice Problem** for students to try so that they can check their understanding of the concept covered. Answers to the Practice Problems are provided in the back of the book.

Procedure in Action Examples. These types of examples, interspersed throughout the text, present important procedures in numbered steps. Special **Procedure in Action** examples present important multistep procedures, such as the steps for doing synthetic division, in a two-column format. The steps of the procedure are given in the left column, and an example is worked, following these steps, in the right column. This approach provides students with a clear model with which they can compare when encountering difficulty in their work.

Additional Pedagogical Features

Definitions, ***Theorems***, ***Properties***, and ***Rules*** are all boxed and titled for emphasis and ease of reference.

Warnings appear as appropriate throughout the text to apprise students of common errors and pitfalls that can trip them up in their thinking or calculations.

Summary of Main Facts boxes summarize information related to equations and their graphs, such as those of the conic sections.

A Calculus Symbol appears next to information in the text that is essential for the study of calculus.

Margin Notes

Side Notes provide hints for handling newly introduced concepts.

Recall notes remind students of a key idea learned earlier in the text that will help them work through a current problem.

Technology Connections give students tips on using calculators to solve problems, check answers, and reinforce concepts. Note that the use of graphing calculators is optional in this text.

Do You Know? Features provide students with additional interesting information on topics to keep them engaged in the mathematics presented.

Exercises. The heart of any textbook is its exercises, so we have tried to ensure that the quantity, quality, and variety of exercises meet the needs of all students. Exercises are carefully graded to strengthen the skills developed in the section and are organized using the following categories.

Concepts and Vocabulary exercises begin each exercise set with problems that assess the student's grasp of the definitions and ideas introduced in that section. These true-false and fill-in-the-blank exercises help to rapidly identify gaps in comprehension of the material in that section.

Building Skills exercises develop fundamental skills—each odd-numbered exercise is closely paired with its consecutive even-numbered exercise.

Applying the Concepts exercises use the section's material to solve real-world problems—all are titled and relevant to the topics of the section.

Beyond the Basics exercises provide more challenging problems that give students an opportunity to reach beyond the material covered in the section—these are generally more theoretical in nature and are suitable for honors students, special assignments, or extra credit.

Critical Thinking/Discussion/Writing exercises, appearing as appropriate, are designed to develop students' higher-level thinking skills. Calculator problems, identified by , are included where needed.

Active Learning exercises allow students to explore mathematical concepts in new ways. Students have the chance to manipulate Interactive Figures and answer accompanying questions.

Key Ideas at a Glance. This one page feature found at the end of each chapter highlights some key concepts in each chapter. In some chapters, this will serve as a comparison between two similar or parallel topics. In other chapters, this will sum up many of the ideas presented in the chapter. This page can serve as a reference to students to look back on when studying or doing exercises. There are also exercises to accompany this feature so that students may test their understanding of the ideas summarized there.

Chapter Review and Tests. The chapter-ending material begins with an extensive **Review** featuring a two-column, section-by-section summary of the definitions, concepts, and formulas covered in that chapter, with corresponding examples. This review provides a description and examples of key topics indicating where the material occurs in the text, and encourages students to reread sections rather than memorize definitions out of context. **Review Exercises** provide students with an opportunity to practice what they have learned in the chapter. Then students are given two chapter test options. They can take **Practice Test A** in the usual open-ended format and/or **Practice Test B**, covering the same topics, in a multiple-choice format. Practice Test B has been moved online for this edition, and can be found in the eText. All tests are designed to increase student comprehension and verify that students have mastered the skills and concepts in the chapter. Mastery of these materials should indicate a true comprehension of the chapter and the likelihood of success on the associated in-class examination. **Cumulative Review Exercises** appear at the end of every chapter, starting with Chapter 2, to remind students that mathematics is not modular and that what is learned in the first part of the book will be useful in later parts of the book and on the final examination.

MyLab Math Resources for Success

MyLab Math is available to accompany Pearson's market-leading text options, including ***Precalculus: A Right Triangle Approach*, 5th Edition** (access code required).

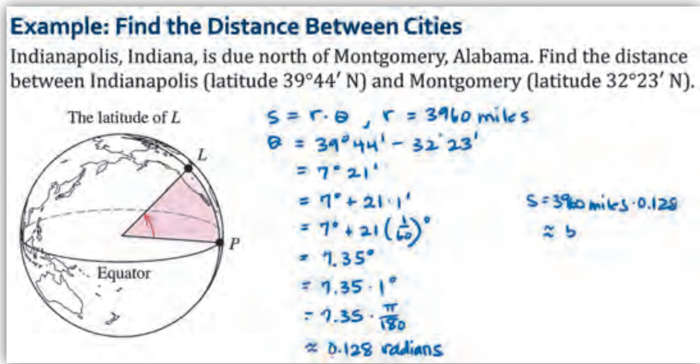
MyLab™ is the teaching and learning platform that empowers you to reach every student. MyLab Math combines trusted author content—including full eText and assessment with immediate feedback—with digital tools and a flexible platform to personalize the learning experience and improve results for each student.

MyLab Math supports all learners, regardless of their ability and background, in order to provide an equal opportunity for success. Accessible resources support learners for a more equitable experience no matter their abilities. And options to personalize learning and address individual gaps help to provide each learner with the specific resources they need in order to achieve success.

Student Resources

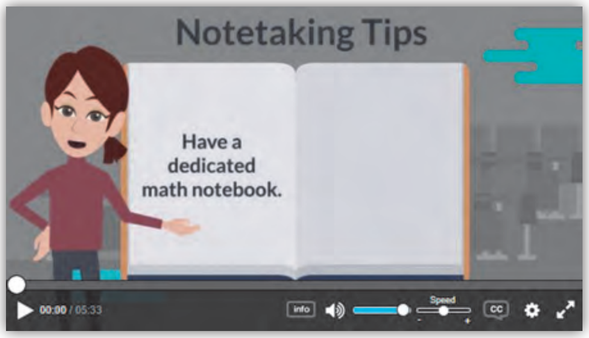
Motivate Your Students—Students are motivated to succeed when they're engaged in the learning experience and understand the relevance and power of math.

▼ **NEW!** **Section Lecture Videos**—Co-authors Jessica Bernards and Wendy Fresh (Portland Community College) have created all new Section videos, segmented and assignable by objective, using their years of teaching experience for online courses and flipped classrooms. Instructors can assign a full objective or only the segment that is needed. The videos allow students an opportunity to learn from experienced master teachers breaking down complex topics in an easy-to-understand manner.



- **NEW!** **Video Notebook** is a note-taking guide that gives students a structured place to take notes and work the example problems as they watch the videos. Definitions and important concepts are highlighted, helpful tips are pointed out along the way. Jessica Bernards and Wendy Fresh author this supplement to make sure students are actively engaged with their learning. The Video Notebook is available as PDFs and customizable Word files in MyLab Math. Instructors can also use the video notebook as a guide when creating their own lecture notes in a traditional lecture class. This way an instructor has pre-built lecture notes ready to go, or an easily adaptable set of lecture notes ready to be modified for the needs of their students.

▼ **NEW!** **Mathematical study skills videos**, created by co-authors Jessica Bernards and Wendy Fresh, motivate students to stick with their math course and offer practical tips to succeed. The animated character, Polly Nomial, guides students through topics such as How Learning Math is Different and Having a Growth Mindset in Math that any math student could benefit from watching. These ten study skills videos have pre-built assignments that include assessment questions that test students' understanding of the content.



- **NEW!** **Personal Inventory Assessments** are a collection of online exercises designed to promote self-reflection and metacognition in students. These 33 assessments include topics such as a Stress Management Assessment, Diagnosing Poor Performance and Enhancing Motivation, and Time Management Assessment.

Address Underpreparedness—Each student learns at a different pace. Personalized learning pinpoints the precise areas where each student needs practice, giving all students the support they need—when and where they need it—to be successful.

NEW! **Integrated Review** can be used in corequisite courses, or simply to help students who enter Precalculus without a full understanding of prerequisite skills and concepts.

- Integrated Review at the chapter level provides a Skills Check assessment to pinpoint which prerequisites topics, if any, students need to review.
- Students who require additional review proceed to a personalized homework assignment to remediate.
- Integrated Review videos and worksheets provide additional instruction.

Instructors who prefer to review at the section level can assign the Enhanced Assignments instead.

Personalized Homework—With Personalized Homework, students take a quiz or test and receive a subsequent homework assignment that is personalized based on their performance. This way, students can focus on just the topics they have not yet mastered.

Other student resources include the following:

- **NEW!** **Interactive Figures** bring mathematical concepts to life, helping students see the concepts through directed explorations and purposeful manipulation. For this revision, we added many more interactive figures (in editable GeoGebra format) to the Video & Resource Library. The instructional videos that accompany the text now include Interactive Figures to teach key concepts. These figures are assignable in MyLab Math and encourage active learning, critical thinking, and conceptual understanding.
- **Solution Manual**—Written by Beverly Fusfield, the Student's Solution Manual provides detailed worked-out solutions to the odd-numbered end-of-section and Chapter Review exercises as well as solutions to all the Practice Problems, Practice Tests, and Cumulative Review problems. Available in MyLab Math.

Instructor Resources

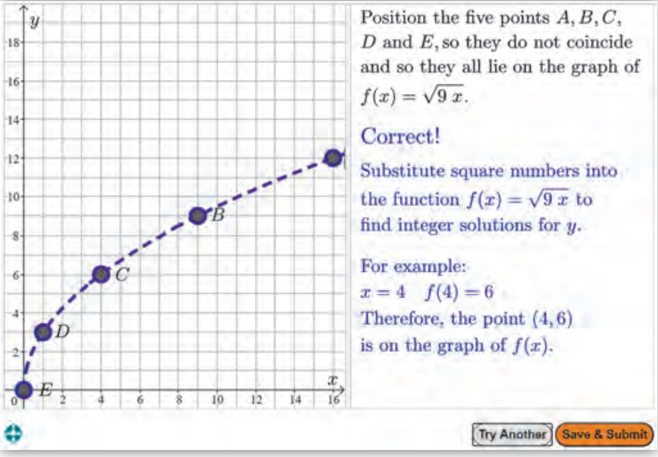
Your course is unique. So whether you'd like to build your own assignments, teach multiple sections, or set prerequisites, MyLab gives you the flexibility to easily create your course to fit your needs.

Pre-Built Assignments are designed to maximize students' performance. All assignments are *fully editable* to make your course your own.

- **NEW!** **Enhanced Assignments**—These section-level assignments have three unique properties:
  1. They help keep skills fresh with *spaced practice* of previously learned concepts.
  2. Learning aids are strategically turned off for some exercises to ensure students understand how to work the exercises independently.
  3. They contain personalized prerequisite skills exercises for gaps identified in the chapter-level Skills Check Quiz.
- **NEW!** **Learning Assignments**—Section-level assignments are especially helpful for online classes or flipped classes, where some or all learning takes place independently. These assignments include objective-level videos and interactive figures for student exploration followed by corresponding MyLab questions to ensure engagement and understanding. Instructors can assign the video notebook for students to fill out as they complete these video assignments.

MyLab Math Question Library is correlated to the exercises in the text, reflecting each author's approach and learning style. They regenerate algorithmically to give students unlimited opportunity for practice and mastery. Below are a few exercise types available to assign:

- **NEW!** **GeoGebra Exercises** are gradable graphing and computational exercises that help students demonstrate their understanding. They enable students to interact directly with the graph in a manner that reflects how students would graph on paper.



- **Setup & Solve Exercises** require students to first describe how they will set up and approach the problem. This reinforces conceptual understanding of the process applied in approaching the problem, promotes long-term retention of the skill, and mirrors what students will be expected to do on a test.
- **Concept and Vocabulary**—Each exercise section begins with exercises that assess the student's grasp of the definitions and ideas introduced in that section. These true-false and fill-in-the-blank exercises help to rapidly identify gaps in comprehension and are assignable in MyLab Math and Learning Catalytics.

Learning Catalytics—With Learning Catalytics, you'll hear from every student when it matters most. You pose a variety of questions in class (choosing from pre-loaded questions or your own) that help students recall ideas, apply concepts, and develop critical-thinking skills. Your students respond using their own smartphones, tablets, or laptops.

Performance Analytics enable instructors to see and analyze student performance across multiple courses. Based on their current course progress, individuals' performance is identified above, at, or below expectations through a variety of graphs and visualizations.

Now included with Performance Analytics, **Early Alerts** use predictive analytics to identify struggling students—even if their assignment scores are not a cause for concern. In both Performance Analytics and Early Alerts, instructors can email students individually or by group to provide feedback.

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Other instructor resources include the following:

- **Instructor Solution Manual**—Written by Bevery Fusfield, the Instructor's Solutions Manual provides complete solutions for all end-of-section exercises, including the Critical Thinking/Discussion/Writing Projects, Practice Problems, Chapter Review exercises, Practice Tests, and Cumulative Review problems.
- **PowerPoint Lecture Slides** feature presentations written and designed specifically for this text, including figures and examples from the text. Accessible versions of the PowerPoints are also available.
- **TestGen** enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same questions or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download at pearson.com.
- **Test Bank** features a printable PDF containing all the test exercises available in TestGen. The current version contains 6 forms of tests per chapter in PDF format. Forms A-D are open-ended. Forms E and F are multiple choice.

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We invite all who use this book to send suggestions for improvements to Marcus McWaters at mmm@usf.edu.

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DEDICATION

To Our Spouses,

Lata, Debra, Leslie, Kevin, and Jon

P Basic Concepts of Algebra

TOPICS

P.1 The Real Numbers and Their Properties

P.2 Integer Exponents and Scientific Notation

P.3 Polynomials

P.4 Factoring Polynomials

P.5 Rational Expressions

P.6 Rational Exponents and Radicals

Many fascinating patterns in human and natural processes can be described in the language of algebra. We investigate events ranging from chirping crickets to the behavior of falling objects.

P.1 The Real Numbers and Their Properties

Videos for this Section

Objectives

1 ▸ Classify sets of real numbers.
2 ▸ Use exponents.
3 ▸ Use the ordering of the real numbers.
4 ▸ Specify sets of numbers in roster or set-builder notation.
5 ▸ Use interval notation.
6 ▸ Relate absolute value and distance on the real number line.
7 ▸ Use the order of operations in arithmetic expressions.
8 ▸ Identify and use properties of real numbers.
9 ▸ Evaluate algebraic expressions.

Cricket Chirps and Temperature

Crickets are sensitive to changes in air temperature; their chirps speed up as the temperature gets warmer and slow down as it gets cooler. It is possible to use the chirps of the male snowy tree cricket (*Oecanthus fultoni*), common throughout the United States, to gauge temperature. (The insect is found in every U.S. state except Hawaii, Alaska, Montana, and Florida.) By counting the chirps of this cricket, which lives in bushes a few feet from the ground, you can gauge temperature. Snowy tree crickets are more accurate than most cricket species; their chirps are slow enough to count, and they synchronize their singing. To convert cricket chirps to degrees Fahrenheit, count the number of chirps in 14 seconds and then add 40 to get the temperature. To convert cricket chirps to degrees Celsius, count the number of chirps in 25 seconds, divide by 3, and then add 4 to get the temperature. In Example 12, we evaluate algebraic expressions to learn the temperature from the number of cricket chirps.

Objective 1 ▸

Classifying Numbers

In algebra, we use letters such as a, b, x, y, and so on, to represent numbers. A letter that is used to represent one or more numbers is called a **variable**. A **constant** is a specific number such as 3 or $\frac{1}{2}$ or a letter that represents a fixed (but not necessarily specified) number. Physicists use the letter c as a constant to represent the speed of light ($c \approx 300{,}000{,}000$ meters per second).

SIDE NOTE

Here is one difficulty with attempting to divide by 0: If, for example, $\frac{5}{0} = a$, then $5 = a \cdot 0$. However, $a \cdot 0 = 0$ for all numbers a. So we would have $5 = 0$; this contradiction demonstrates that there is no appropriate choice for $\frac{5}{0}$.

We use two variables, a and b, to denote the results of the operations of addition $(a + b)$, subtraction $(a - b)$, multiplication $(a \times b$ or $a \cdot b)$, and division $\left(a \div b \text{ or } \frac{a}{b}\right)$. These operations are called **binary operations** because each is performed on two numbers.

We frequently omit the multiplication sign when writing a product involving two variables (or a constant and a variable) so that $a \cdot b$ and ab indicate the same product. Both a and b are called **factors** in the product $a \cdot b$. This is a good time to recall that we never divide by zero. For $\frac{a}{b}$ to represent a real number, b cannot be zero.

Equality of Numbers

The **equal sign**, $=$, is used much like we use the word *is* in English. The equal sign means that the number or expression on the left side is equal or equivalent to the number or expression on the right side. We write $a \neq b$ to indicate that a is not equal to b.

Classifying Sets of Numbers

The idea of a set is familiar to us. We regularly refer to "a set of baseball cards," a "set of CDs," or "a set of dishes." In mathematics, as in everyday life, a **set** is a collection of objects. The objects in the set are called the **elements**, or **members**, of the set. Capital letters are usually used to name a set. In the study of algebra, we are interested primarily in sets of numbers.

In listing the elements of a set, it is customary to enclose the listed elements in braces, { }, and separate them with commas.

We distinguish among various sets of numbers.

The numbers we use to count with constitute the set N of **natural numbers**: $N = \{1, 2, 3, 4, \ldots\}$.

The three dots . . . (called ellipsis) may be read as "and so on" and indicate that the pattern continues indefinitely.

The set W of **whole numbers** is formed by including the number 0 with the natural numbers to obtain the set: $W = \{0, 1, 2, 3, 4, \ldots\}$.

The set Z of **integers** consists of the set N of natural numbers together with their opposites and 0: $Z = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$.

Rational Numbers

The **rational numbers** consist of all numbers that *can* be expressed as the quotient or ratio, $\frac{a}{b}$, of two integers, where $b \neq 0$. The letter Q is often used to represent the set of rational numbers.

Examples of rational numbers are $\frac{1}{2}, \frac{5}{3}, \frac{-4}{17}$, and $0.07 = \frac{7}{100}$. Any integer a can be expressed as the quotient of two integers by writing $a = \frac{a}{1}$. Consequently, every integer is also a rational number. In particular, 0 is a rational number because $0 = \frac{0}{1}$.

The rational number $\frac{a}{b}$ can be written as a decimal by using long division. When any integer a is divided by an integer b, $b \neq 0$, the result is always a **terminating decimal** such as $\frac{1}{2} = 0.5$ or a **nonterminating repeating decimal** such as $\frac{2}{3} = 0.666\ldots$.

We sometimes place a bar over the repeating digits in a nonterminating repeating decimal. Thus, $\frac{2}{3} = 0.666\ldots = 0.\overline{6}$ and $\frac{141}{110} = 1.2818181\ \ldots = 1.2\overline{81}$.

EXAMPLE 1 Converting Decimal Rationals to a Quotient

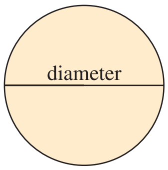
Write the rational number $7.\overline{45}$ as the ratio of two integers in lowest terms.

Solution

Let $x = 7.454545 \ldots$. Then

$$\begin{aligned} 100x &= 745.4545\ldots && \text{Multiply both sides by 100.} \\ \text{subtract} \quad x &= 7.4545\ldots \\ \hline 99x &= 738 && 100x - x = 99x \\ x &= \frac{738}{99} && \text{Divide both sides by 99.} \\ &= \frac{82 \times \cancel{9}}{11 \times \cancel{9}} && \text{Common factor} \\ x &= \frac{82}{11}. && \text{Reduce to lowest terms.} \end{aligned}$$

Practice Problem 1. Repeat Example 1 for $2.132132132\ldots$. ■



$\pi = \dfrac{\text{circumference}}{\text{diameter}}$

Figure P.1 ▸ Definition of π

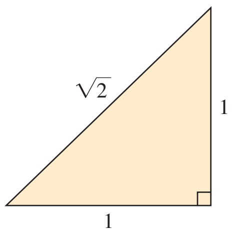


Figure P.2

RECALL

An integer is a *perfect square* if it is a product $a \cdot a$, where a is an integer. For example, $9 = 3 \cdot 3$ is a perfect square.

Irrational Numbers

An **irrational number** is a real number that cannot be written as a ratio of two integers. This means that its decimal representation must be nonrepeating and nonterminating. We can construct such a decimal using only the digits 0 and 1, such as $0.01001000100001\ldots$. Because each group of zeros contains one more zero than the previous group, no group of digits repeats. Other numbers such as π ("pi"; see Figure P.1) and $\sqrt{2}$ (the square root of 2; see Figure P.2) can also be expressed as decimals that neither terminate nor repeat; so they are irrational numbers as well. We can obtain an approximation of an irrational number by using an initial portion of its decimal representation. For example, we can write $\pi \approx 3.14159$ or $\sqrt{2} \approx 1.41421$, where the symbol $\approx$ is read "is approximately equal to."

No familiar process, such as long division, is available for obtaining the decimal representation of an irrational number. However, your calculator can provide a useful approximation for irrational numbers such as $\sqrt{2}$. (Try it!) Because a calculator displays a fixed number of decimal places, it gives a **rational approximation** of an irrational number.

It is usually not easy to determine whether a specific number is irrational. One helpful fact in this regard is that *the square root of any natural number that is not a perfect square is irrational.* So $\sqrt{6}$ is irrational but $\sqrt{16} = \sqrt{4^2} = 4$ is rational.

Because rational numbers have decimal representations that either terminate or repeat, whereas irrational numbers do not have such representations, *no number is both rational and irrational.*

The rational numbers together with the irrational numbers form the set R of **real numbers**.

The diagram in Figure P.3 shows how various sets of numbers are related. For example, every natural number is also a whole number, an integer, a rational number, and a real number.

Real Numbers

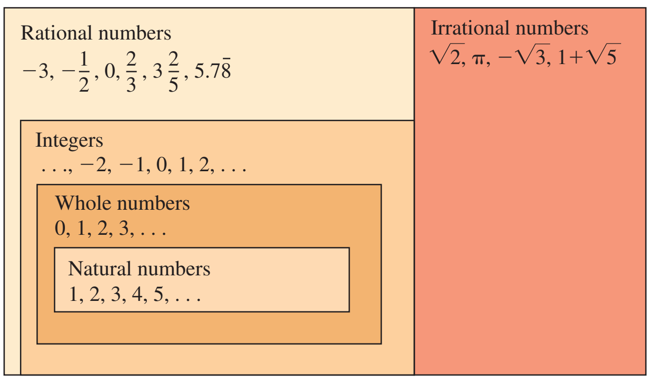


Figure P.3 ▸ Relationships among sets of real numbers

EXAMPLE 2 Identifying Sets of Numbers

Let $A = \left\{-17, -5, -\frac{6}{3}, -\frac{2}{3}, 0, \frac{5}{12}, \frac{1}{2}, \sqrt{2}, \pi, \sqrt{35}, 7, 18\right\}$.

Identify all the elements of the set A that are

a. Natural numbers **b.** Whole numbers **c.** Integers
d. Rational numbers **e.** Irrational numbers **f.** Real numbers

Solution

a. Natural numbers: 7 and 18

b. Whole numbers: 0, 7, and 18

c. Integers: $-17, -5, -\frac{6}{3}$ (or -2), 0, 7, and 18

d. Rational numbers: $-17, -5, -\frac{6}{3}, -\frac{2}{3}, 0, \frac{5}{12}, \frac{1}{2}$, 7, and 18

e. Irrational numbers: $\sqrt{2}$, π, and $\sqrt{35}$

f. Real numbers: All numbers in the set A are real numbers.

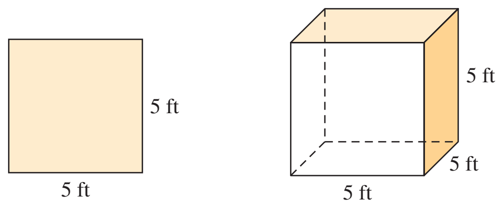
Practice Problem 2. Repeat Example 2 for the following set:

$B = \left\{-6, -\frac{21}{7}, -\frac{1}{2}, 0, \frac{4}{3}, \sqrt{3}, 2, \sqrt{17}, 7\right\}$. ■

Objective 2 ▸

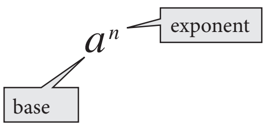
Integer Exponents

The area of a square with sides of five feet each is $5 \cdot 5 = 25$ square feet.
The volume of a cube that has sides of 5 feet each is $5 \cdot 5 \cdot 5 = 125$ cubic feet.



Area = 5·5 square feet Volume = 5·5·5 cubic feet

A shorter notation for $5 \cdot 5$ is 5^2 and for $5 \cdot 5 \cdot 5$ is 5^3. The number 5 is called the *base* for both 5^2 and 5^3. The number 2 is called the *exponent* in the expression 5^2 and indicates that the base 5 appears as a factor twice.



Positive Integer Exponent

If a is a real number and n is a positive integer, then

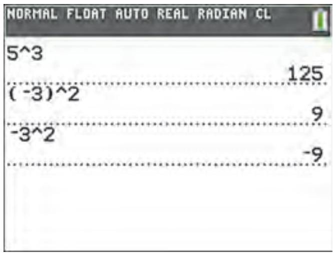
$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}}.$$

The number a^n is called the nth power of a and is read "a to the nth power," or "a to the n." The number a is called the **base**; n, the **exponent**. We adopt the convention that $a^1 = a$.

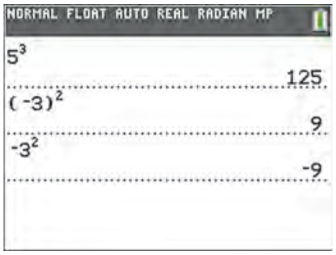
TECHNOLOGY CONNECTION

Type 5^3 for 5^3 on a graphing calculator. Any expression on a calculator enclosed in parentheses and followed by ^n is raised to the nth power.

A common error is to forget parentheses when computing an expression such as $(-3)^2$, with the result being -3^2. Newer graphing calculators, such as the TI-84 series, provide display options called CLASSIC and MATHPRINT. The MATHPRINT option displays 5^3 as 5^3 but still requires you to type 5, then ^, then 3.



CLASSIC Mode



MATHPRINT Mode

In this text we will display screens in MATHPRINT mode.

EXAMPLE 3 Evaluating Expressions That Use Exponents

Evaluate each expression.

a. 5^3 **b.** $(-3)^2$ **c.** -3^2 **d.** $(-2)^3$

Solution

a. $5^3 = 5 \cdot 5 \cdot 5 = 125$ 5 is the base, and 3 is the exponent.

b. $(-3)^2 = (-3)(-3) = 9$ $(-3)^2$ is the opposite of 3, squared.

c. $-3^2 = -3 \cdot 3 = -9$ Multiplication of $3 \cdot 3$ occurs first. (-3^2 is the opposite of 3^2.)

d. $(-2)^3 = (-2)(-2)(-2) = -8$

Practice Problem 3. Evaluate the following.

a. 2^3 **b.** $(3a)^2$ **c.** $\left(\frac{1}{2}\right)^4$ ■

In Example 3, pay careful attention to the fact (from parts **b** and **c**) that $(-3)^2 \neq -3^2$. In $(-3)^2$, the parentheses indicate that the exponent 2 applies to the base -3, whereas in -3^2, the absence of parentheses indicates that the exponent applies only to the base 3. When n is even, $(-a)^n \neq -a^n$ for $a \neq 0$.

The Real Number Line

We associate the real numbers with points on a geometric line (imagined to be extended indefinitely in both directions) in such a way that each real number corresponds to exactly one point and each point corresponds to exactly one real number. The point is called the **graph** of the corresponding real number, and the real number is called the **coordinate** of the point. By agreement, ***positive numbers*** lie to the right of the point corresponding to 0 and ***negative numbers*** lie to the left of 0. See Figure P.4.

Notice that $\frac{1}{2}$ and $-\frac{1}{2}$, 2 and -2, and π and $-\pi$ correspond to pairs of points exactly the same distance from 0 but on opposite sides of 0.

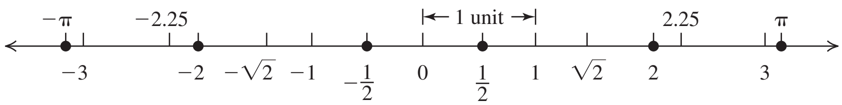


Figure P.4 ▸ The real number line

When coordinates have been assigned to points on a line in the manner just described, the line is called a **real number line**, a **coordinate line**, a **real line**, or simply a **number line**. The point corresponding to 0 is called the **origin**.

Objective 3 ▸

Inequalities

The real numbers are **ordered** by their size. We say that ***a* is less than *b*** and write $a < b$, provided that $b = a + c$ for some ***positive*** number c. We also write $b > a$, meaning the same thing as $a < b$, and say that ***b* is greater than *a***. On the real line, the numbers increase from left to right. Consequently, ***a is to the left of b on the number line when a < b***. Similarly, a is to the right of b on the number line when $a > b$. We sometimes want to indicate that at least one of two conditions is correct: Either $a < b$ or $a = b$. In this case, we write $a \leq b$ or $b \geq a$. The four symbols $<, >, \leq$, and $\geq$ are called **inequality symbols**.

SIDE NOTE

Notice that the inequality sign always points to the smaller number.

$2 < 7$, 2 is smaller.

$5 > 1$, 1 is smaller.

EXAMPLE 4 Identifying Inequalities

Decide whether each of the following is true or false from their position on a number line.

a. $5 > 0$ **b.** $-2 < -3$ **c.** $2 \leq 3$ **d.** $4 \leq 4$

Solution

a. $5 > 0$ is true because 5 is to the right of 0 on the number line. See Figure P.5.

b. $-2 < -3$ is false because -2 is to the right of -3 on the number line.

c. $2 \leq 3$ is true because 2 is to the left of 3 on the number line. (Recall that $2 \leq 3$ is true if either $2 < 3$ or $2 = 3$.)

d. $4 \leq 4$ is true because $4 \leq 4$ is true if either $4 < 4$ or $4 = 4$.

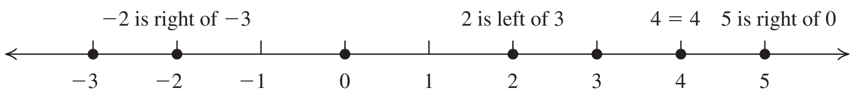


Figure P.5

Practice Problem 4. Decide whether each of the following is true or false.

a. $-2 < 0$ **b.** $5 \leq 7$ **c.** $-4 > -1$ ■

The following properties of inequalities for real numbers are used throughout this text. Here a, b, and c represent real numbers.

Trichotomy Property: Exactly one of the following is true:

$$a < b,\ a = b, \text{ or } a > b.$$

Transitive Property: If $a < b$ and $b < c$, then $a < c$.

The trichotomy property says that if two real numbers are not equal, then one is larger than the other. The transitive property says that "less than" works like "smaller than" or "lighter than." Frequently, we read $a > 0$ as "***a* is positive**" instead of "a is greater than 0." We can also read $a < 0$ as "***a* is negative**." If $a \geq 0$, then either $a > 0$ or $a = 0$, and we may say that "***a* is nonnegative**."

Objective 4 ▸

Sets

To specify a set, we do one of the following:

1. List the elements of the set (**roster method**).
2. Describe the elements of the set (often using **set-builder notation**).

Variables are helpful in describing sets when we use set-builder notation. The notation $\{x | x$ is a natural number less than $6\}$ is in set-builder notation and describes the set $\{1, 2, 3, 4, 5\}$ using the roster method.

We read $\{x | x$ is a natural number less than $6\}$ as "the set of all x such that x is a natural number less than six." Generally, $\{x | x$ has property $P\}$ designates the set of all x such that (the vertical bar is read "such that") x has property P.

It may happen that a description fails to describe any number. For example, consider $\{x | x < 2 \text{ and } x > 7\}$. Of course, no number can be simultaneously less than 2 and greater than 7, so this set has no members. We refer to a set with no elements as the **empty set**, or **null set**, and use the special symbol $\varnothing$, or sometimes $\{\ \}$, to denote it.

Definition of Union and Intersection

The **union** of two sets A and B, denoted $A \cup B$, is the set consisting of all elements that are in A *or* B (or both). See Figure P.6a. The **intersection** of A and B, denoted $A \cap B$, is the set consisting of all elements that are in both A *and* B. In other words, $A \cap B$ consists of the elements common to A and B. See Figure P.6b.

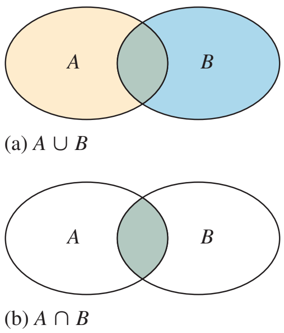


Figure P.6 ▸ Picturing union and intersection

EXAMPLE 5 Forming Set Unions and Intersections

Find $A \cap B$, $A \cup B$, and $A \cap C$, if $A = \{-2, -1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2, 4\}$, and $C = \{-3, 3\}$.

Solution

$A \cap B = \{-2, 0, 2\}$, the set of elements common to both A and B.

$A \cup B = \{-4, -2, -1, 0, 1, 2, 4\}$, the set of elements that are in A or B (or both).

$A \cap C = \varnothing$

Practice Problem 5. Find $A \cap B$ and $A \cup B$, if $A = \{-3, -1, 0, 1, 3\}$ and $B = \{-4, -2, 0, 2, 4\}$. ■

Objective 5 ▸

Intervals

We now turn our attention to graphing certain sets of numbers. That is, we graph each number in a given set. We are particularly interested in sets of real numbers, called **intervals**, whose graphs correspond to special sections of the number line.

If $a < b$, then the set of real numbers between a and b, but not including either a or b, is called the **open interval** from a to b and is denoted by (a, b). See Figure P.7. Using set-builder notation, we can write

$$(a, b) = \{x | a < x < b\}.$$

$(a, b) = \{x \mid a < x < b\}$

Figure P.7 ▸ An open interval

We indicate graphically that the endpoints a and b are excluded from the open interval by drawing a left parenthesis at a and a right parenthesis at b. These parentheses enclose the numbers between a and b.

The **closed interval** from a to b is the set

$$[a, b] = \{x | a \le x \le b\}.$$

$[a, b] = \{x \mid a \le x \le b\}$

Figure P.8 ▸ A closed interval

The closed interval includes both endpoints a and b. We replace the parentheses with square brackets in the interval notation and on the graph. See Figure P.8. Sometimes we want to include only one endpoint of an interval and exclude the other. Table P.1 below shows how this is done.

An alternative notation for indicating whether endpoints are included uses open circles to show exclusion and closed circles to show inclusion. See Figure P.9.

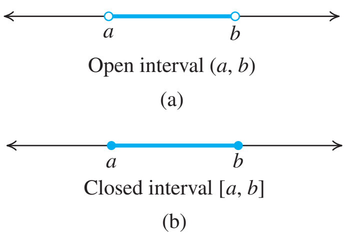


Figure P.9 ▸ Endpoint inclusion and exclusion

If an interval extends indefinitely in one or both directions, it is called an **unbounded interval**. For example, the set of all numbers to the right of 2,

$$\{x | x > 2\},$$

is an unbounded interval denoted by $(2, \infty)$. See Figure P.10.

Interval $(2, \infty)$

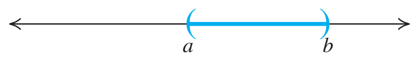
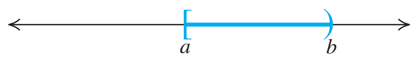
Figure P.10 ▸ An unbounded interval

The symbol ∞ (“infinity”) is not a number, but is used to indicate all numbers to the right of 2.

The symbol $-\infty$ is another symbol that does not represent a number. The notation $(-\infty, a)$ is used to indicate the set of all real numbers that are less than a. The notation $(-\infty, \infty)$ represents the set of all real numbers.

Table P.1 lists various types of intervals that we use in this text. In the table, when two points a and b are given, we assume that $a < b$. This is because if $a > b$, then (a, b) is the empty set.

TABLE P.1

Interval Notation	Set Notation	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \le x \le b\}$	
$(a, b]$	$\{x \mid a < x \le b\}$	
$[a, b)$	$\{x \mid a \le x < b\}$	
(a, ∞)	$\{x \mid x > a\}$	
$[a, \infty)$	$\{x \mid x \ge a\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \le b\}$	
$(-\infty, \infty)$	$\{x \mid x \text{ is a real number}\}$	

SIDE NOTE

The symbols ∞ and $-\infty$ are always used with parentheses, not square brackets. Also note that $<$ and $>$ are used with parentheses and that $\le$ and $\ge$ are used with square brackets.

EXAMPLE 6 Union and Intersection of Intervals

Consider the two intervals $I_1 = (-3, 4)$ and $I_2 = [2, 6]$.

Find: **a.** $I_1 \cup I_2$. **b.** $I_1 \cap I_2$.

Solution

Use the to (exclude -3.

Use the] to include 6.

a. From Figure P.11, we see that $I_1 \cup I_2 = (-3, 6]$. We note that every number in the interval $(-3, 6]$ is in either I_1 or I_2 or in both I_1 and I_2.

b. We see in Figure P.11 that $I_1 \cap I_2 = [2, 4)$. Every number in the interval $[2, 4)$ is in both I_1 and I_2. Notice that while 4 is in I_2, 4 is not in I_1.

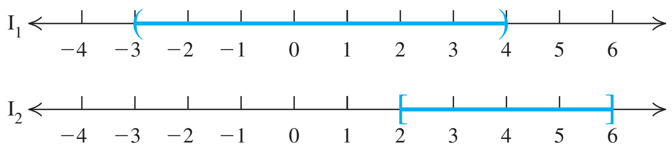


Figure P.11

Practice Problem 6. Let $I_1 = (-\infty, 5)$ and $I_2 = [-2, \infty)$. Find the following.

a. $I_1 \cup I_2$ **b.** $I_1 \cap I_2$ ■

Objective 6 ▸

Absolute Value

The ***absolute value*** of a number a, denoted by $|a|$, is the distance between the origin and the point on the number line with coordinate a. The point with coordinate -3 is 3 units from the origin, so we write $|-3| = 3$ and say that the absolute value of -3 is 3. See Figure P.12.

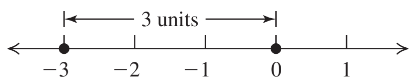


Figure P.12 ▸ Absolute value