GLOBAL EDITION

THOMAS' CALCULUS

Early Transcendentals

FIFTEENTH EDITION IN SI UNITS

Hass • Heil • Bogacki • Weir

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THOMAS'

CALCULUS

Early Transcendentals

FIFTEENTH EDITION IN SI UNITS

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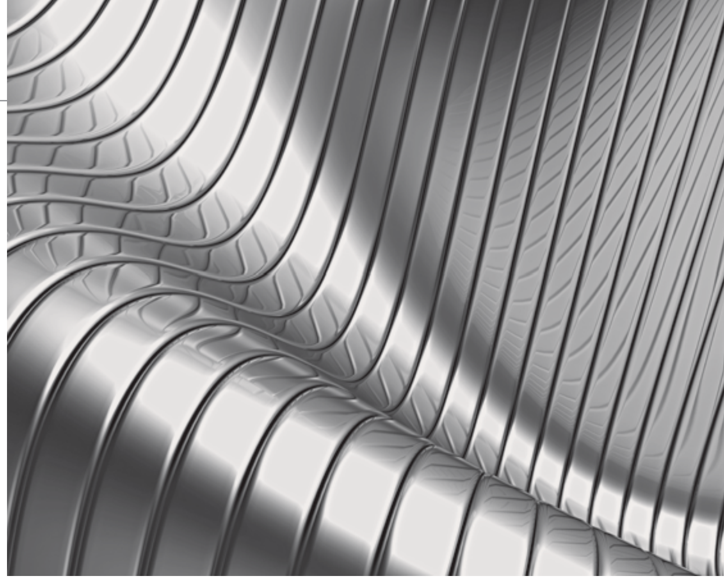
Contents



Preface

Thomas' Calculus: Early Transcendentals, Fifteenth Edition in SI Units, continues its tradition of clarity and precision in calculus with a modern update to the popular text. The authors have worked diligently to add exercises, revise figures and narrative for clarity, and update many applications to modern topics. *Thomas' Calculus* remains a modern and robust introduction to calculus, focusing on developing conceptual understanding of the underlying mathematical ideas. This text supports a calculus sequence typically taken by students in STEM fields over several semesters. Intuitive and precise explanations, thoughtfully chosen examples, superior figures, and time-tested exercise sets are the foundation of this text. We continue to improve this text in keeping with shifts in both the preparation and the goals of today's students, and in the applications of calculus to a changing world.

As Advanced Placement Calculus continues to grow in popularity for high school students, many instructors have communicated mixed reviews of the benefit for today's university and community college students. Some instructors report receiving students with an overconfidence in their computational abilities coupled with underlying gaps in algebra and trigonometry mastery, as well as poor conceptual understanding. In this text, we seek to meet the needs of the increasingly varied population in the calculus sequence. We have taken care to provide enough review material (in the text and appendices), detailed solutions, and a variety of examples and exercises, to support a complete understanding of calculus for students at varying levels. Additionally, the MyLab Math course that accompanies the text provides significant support to meet the needs of all students. Within the text, we present the material in a way that supports the development of mathematical maturity, going beyond memorizing formulas and routine procedures, and we show students how to generalize key concepts once they are introduced. References are made throughout, tying new concepts to related ones that were studied earlier. After studying calculus from *Thomas*, students will have developed problem-solving and reasoning abilities that will serve them well in many important aspects of their lives. Mastering this beautiful and creative subject, with its many practical applications across so many fields, is its own reward. But the real gifts of studying calculus are acquiring the ability to think logically and precisely; understanding what is defined, what is assumed, and what is deduced; and learning how to generalize conceptually. We intend this book to encourage and support those goals.

New to This Edition

We welcome to this edition a new coauthor, Przemyslaw Bogacki from Old Dominion University. Przemek joined the team for the fourth edition of *University Calculus* and now joins the *Thomas' Calculus* team. Przemek brings a keen eye for details as well as significant experience in MyLab Math. Przemek has diligently reviewed every exercise and solution in MyLab Math for mathematical accuracy, fidelity with text methods, and effectiveness for students. He has also recommended nearly 100 new Setup & Solve exercises and improved the sample assignments in MyLab. Przemek has also written the new appendix on Optimization covering determinants, extreme values, and gradient descent.

The most significant update to this 15th edition includes new online chapters on Complex Functions, Fourier Series and Wavelets, and the new appendix on Optimization. These chapters can provide material for students interested in more advanced topics. The details are outlined below in the chapter descriptions.

We have also made the following updates:

- Many updated graphics and figures to bring out clear visualization and mathematical correctness.
- Many wording clarifications and revisions.
- Many instruction clarifications for exercises, such as suggesting where the use of a calculator may be needed.
- Notation of inverse trig functions favoring arcsin notation over $\sin^{-1}$, etc.

New to MyLab Math

Pearson has continued to improve the general functionality of MyLab Math since the previous edition. Ongoing improvements to the grading algorithms, along with the development of MyLab Math for our differential equations courses allows for more sophisticated acceptance of generic constants and better parsing of mathematical expressions.

- The full suite of interactive figures has been updated for accessibility meeting WCAG standards. These figures are designed to be used in lecture as well as by students independently. The figures are editable using the freely available GeoGebra software. The figures were created by Marc Renault (Shippensburg University), Kevin Hopkins (Southwest Baptist University), Steve Phelps (University of Cincinnati), and Tim Brzezinski (Southington High School, CT).
- New! GeoGebra Exercises are gradable graphing and computational exercises that help students demonstrate their understanding. They enable students to interact directly with the graph in a manner that reflects how students would graph on paper.
- Nearly 100 additional Setup & Solve exercises have been created, selected by author Przemyslaw Bogacki. These exercises are designed to focus students on the process of problem solving by requiring them to set up their equations before moving on to the solution.
- Integrated Review quizzes and personalized homework are now built into all MyLab Math courses. No separate Integrated Review course is required.
- New online chapters and sections have exercises available, including exercises for the complex numbers and functions that many users have asked for.

Content Enhancements

Chapter 1

- Section 1.2. Revised Example 4 to clarify the distinction between vertical and horizontal scaling of a graph.
- Section 1.3. Added new Figure 1.46, illustrating a geometric proof of the angle sum identities.

Chapter 2

- Section 2.2. New Example 11, illustrating the use of the Sandwich Theorem, with corresponding new Figure 2.14.
- Section 2.4. New subsection on “Limits at Endpoints of an Interval” added. New Example 2 added, illustrating limits at a boundary point of an interval.
- Section 2.6. Exercises 41–45 on limits involving trigonometric functions moved from Chapter 3.
- Additional and Advanced Exercises. Exercises 31–40 on limits involving trigonometric functions moved from Chapter 3.

Chapter 3

- Section 3.8. Revised Figure 3.36 illustrating the relationship between slopes of graphs of inverse functions.
- Updated differentiation formulas involving exponential and logarithmic functions.
- Expanded Example 5.
- Expanded Example 7 to clarify the computation of the derivative of x^x.
- Added new Exercises 11–14 involving the derivatives of inverse functions.
- Section 3.9. Updated differentiation formulas involving inverse trigonometric functions.
- Added new Example 3 to illustrate differentiating a composition involving the arctangent function.
- Rewrote the introduction to the subsection on the derivative of arcsec x.
- Section 3.10. Updated and improved related rates problem strategies, and correspondingly revised Examples 2–6.

Chapter 4

- Section 4.3. Added new Exercises 69–70.
- Section 4.4. Added new Exercises 107–108.
- Section 4.5. Improved the discussion of indeterminate forms.
- Expanded Example 1.
- Added new Exercises 19–20.
- Section 4.6. Updated and improved strategies for solving applied optimization problems.
- Added new Exercises 33–34.
- Section 4.8. Added Table 4.3 of integration formulas.

Chapter 5

- Section 5.1. The Midpoint Rule and the associated formula for calculating an integral numerically were given a more central role and used to introduce a numerical method.
- Section 5.3. New basic theory Exercise 89. Integrals of functions that differ at one point.
- Section 5.6. New Exercises 113–116. Compare areas using graphics and computation.

Chapter 6

Section 6.2. Discussion of cylinders in Example 1 clarified.

Chapter 7

- Clarified derivative formulas involving x versus those involving a differentiable function u.
- Section 7.1. Rewrote material on Logarithms and Laws on Exponents. Exercises 63–66 moved from Chapter 4. New Exercise 67 added.

Chapter 8

- Section 8.3. Clarified computing integrals involving powers of sines and cosines. Exercise 42 replaced. Exercises 51 and 52 added.
- Section 8.4. Ordering of exercises was updated.
- Section 8.5. Discussion of the method of partial fractions rewritten and clarified.
- Section 8.7. New subsection on the Midpoint Rule added. Discussion of Error Analysis expanded to include the Midpoint Rule. Exercises 1–10 expanded to include the Midpoint Rule.
- Section 8.8. Discussion of infinite limits of integration clarified. Material on Tests for Convergence and Divergence, including the Direct Comparison Test and the Limit Comparison Test, their proofs, and associated examples, all revised. New Exercises 69–80 added.

Chapter 9

- Section 9.2. Solution to Example 2 replaced. Solution to Example 8 replaced.
- Section 9.3. Solution to Example 5 revised.

- Section 9.5. Exercise 71 added.
- Section 9.6. Proof of Theorem 15 replaced. Discussion of Theorem 16 revised.
- Section 9.7. Discussion of absolute convergence added to the solution of Example 3. Figure 9.21 revised. New Exercises 40–41 added. Exercise 66 entirely rewritten.
- Section 9.8. Ordering of Exercises was revised. New Exercises 47 and 52 added.
- Section 9.9. Discussion of Taylor series between Examples 4 and 5 rewritten.
- Section 9.10. Exercise 9 replaced.
- Practice Exercises. New Exercises 45–46 added.
- Additional and Advanced Exercises. New Exercises 30–31 added.

Chapter 11

- Section 11.2. New subsection on Vectors in n Dimensions added, with corresponding new Figure 11.19, and new Exercises 60–65.
- Section 11.3. New subsection on The Dot Product of Two n-Dimensional Vectors added, with new Example 9, and new Exercises 53–56.
- Section 11.6. Discussion of cylinders revised.

Chapter 12

Section 12.5. New Exercises 1–2 and 5–6 added.

Chapter 13

- Section 13.2. Added a Composition Rule to Theorem 1 and expanded Example 1.
- Section 13.3. Rewrote the concept of differentiability for functions of several variables to improve clarity.
- Expanded Example 8.
- Section 13.4. Added new Exercises 62–63 on the chain rule with multiple variables.
- Section 13.5. Added a new subsection on gradients for Functions of More Than Three Variables.
- A new Example 7 illustrates a gradient of a 3-variable function.
- New Exercises 45–52 involve gradients of functions with several variables.
- Section 13.7. Added a definition of the Hessian matrix.
- Clarified Example 6.
- Section 13.8. Clarified the use of Lagrange Multipliers throughout, with a more explicit discussion of how to use them for finding maxima and minima.

Chapter 14

- Section 14.2. Added discussion of the properties of limits of iterated double integrals.
- Rewrote Exercises 1–8. Added new Exercises 19–26.
- Section 14.5. Added discussion of the properties of limits of iterated triple integrals. Revised and expanded Example 2.
- Section 14.7. Revised Figure 14.55 to clarify the shape of a spherical wedge involved in triple integration.

Chapter 16

- Section 16.2. Added Figure 16.9.
- Section 16.4. Added a new application of the logistic function showing its connection to Machine Learning and Neural Networks. Added New Exercises 21–22 on the Logistic Equation.

Appendices

Rewrote Appendix A.9 to replace the prime notation with the subscript notation.

New Online Appendix B

B.1 Determinants
B.2 Extreme Values and Saddle Points for Functions of More than Two Variables
B.3 The Method of Gradient Descent

This new appendix covers many topics relevant to students interested in Machine Learning and Neural Networks.

New Online Chapter 18—Complex Functions

This new online chapter gives an introduction to complex functions. Section 1 is an introduction to complex numbers and their operations. It replaces Appendix A.9. Section 2 covers limits and continuity for complex functions. Section 3 introduces complex derivatives and Section 4 the Cauchy-Riemann Equations. Section 5 develops the theory of complex series. Section 6 studies the standard functions such as sin z and Log z, and Section 7 ends the chapter by introducing the theory of conformal maps.

New Online Chapter 19—Fourier Series and Wavelets

This new online chapter introduces Fourier series, and then treats wavelets as a more advanced topic.

It has sections on

19.1 Periodic Functions
19.2 Summing Sines and Cosines
19.3 Vectors and Approximation in Three and More Dimensions
19.4 Approximation of Functions
19.5 Advanced Topic: The Haar System and Wavelets

Continuing Features

Rigor The level of rigor is consistent with that of earlier editions. We continue to distinguish between formal and informal discussions and to point out their differences. Starting with a more intuitive, less formal approach helps students understand a new or difficult concept so they can then appreciate its full mathematical precision and outcomes. We pay attention to defining ideas carefully and to proving theorems appropriate for calculus students, while mentioning deeper or subtler issues they would study in a more advanced course. Our organization and distinctions between informal and formal discussions give the instructor a degree of flexibility in the amount and depth of coverage of the various topics. For example, while we do not prove the Intermediate Value Theorem or the Extreme Value Theorem for continuous functions on a closed finite interval, we do state these theorems precisely, illustrate their meanings in numerous examples, and use them to prove other important results. Furthermore, for those instructors who desire greater depth of coverage, in Appendix A.7 we discuss the reliance of these theorems on the completeness of the real numbers.

Writing Exercises Writing exercises placed throughout the text ask students to explore and explain a variety of calculus concepts and applications. In addition, the end of each chapter contains a list of questions for students to review and summarize what they have learned. Many of these exercises make good writing assignments.

End-of-Chapter Reviews and Projects In addition to problems appearing after each section, each chapter culminates with review questions, practice exercises covering the entire chapter, and a series of Additional and Advanced Exercises with more challenging or synthesizing problems. Most chapters also include descriptions of several **Technology Application Projects** that can be worked by individual students or groups of students over a longer period of time. These projects require the use of *Mathematica* or *Maple*, along with pre-made files that are available for download within MyLab Math.

Writing and Applications This text continues to be easy to read, conversational, and mathematically rich. Each new topic is motivated by clear, easy-to-understand examples and is then reinforced by its application to real-world problems of immediate interest to students. A hallmark of this book has been the application of calculus to science and engineering. These applied problems have been updated, improved, and extended continually over the last several editions.

Technology In a course using the text, technology can be incorporated according to the taste of the instructor. Each section contains exercises requiring the use of technology; these are marked with a T if suitable for calculator or computer use, or they are labeled **Computer Explorations** if a computer algebra system (CAS, such as *Maple* or *Mathematica*) is required.

MyLab Math Resources for Success

MyLab™ Math is available to accompany Pearson's market-leading text options, including ***Thomas' Calculus: Early Transcendentals*, 15th Edition in SI Units** (access code required).

MyLab is the teaching and learning platform that empowers you to reach every student. MyLab Math combines trusted author content—including full eText and assessment with immediate feedback—with digital tools and a flexible platform to personalize the learning experience and improve results for each student.

MyLab Math supports all learners, regardless of their ability and background, to provide an equal opportunity for success. Accessible resources support learners for a more equitable experience no matter their abilities. And options to personalize learning and address individual gaps help to provide each learner with the specific resources they need to achieve success.

Student Resources

Pearson eText—The eText is "reflowable" to adapt to use on tablets and smartphones. You can insert your own highlights, notes, and bookmarks. It is also fully accessible using screen-readers. Download the Pearson+ app to access your eText on your smartphone or tablet anytime—even offline.

Study Slides—PowerPoint slides featuring key ideas and examples are available for students within the Video & Resource Library. These slides are compatible with screen readers.

Address Under-Preparedness—Each student learns at a different pace. Personalized learning pinpoints the precise areas where each student needs practice, giving all students the support they need—when and where they need it—to be successful.

New! Integrated Review can be used for just-in-time prerequisite review.

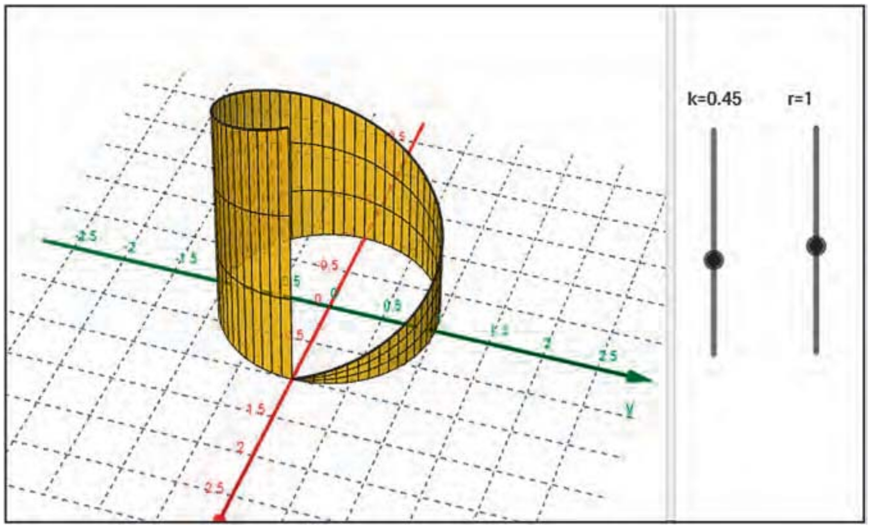
- Integrated Review at the chapter level provides a Skills Check assessment to pinpoint which prerequisite topics, if any, students need to review.
- Students who require additional review proceed to a personalized homework assignment to remediate.
- Integrated Review videos provide additional instruction.

Instructors that prefer to review at the section level can assign the Enhanced Assignments instead.

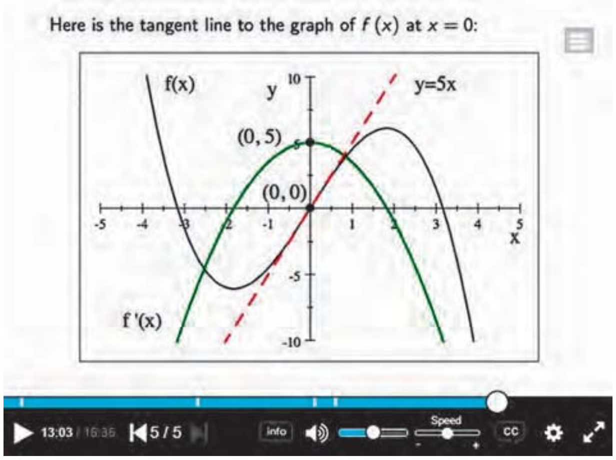
Personalized Homework—With Personalized Homework, students take a quiz or test and receive a subsequent homework assignment that is personalized based on their performance. This way, students can focus on just the topics they have not yet mastered.

Motivate Your Students—Students are motivated to succeed when they're engaged in the learning experience and understand the relevance and power of math.

▼ **Interactive Figures** bring mathematical concepts to life, helping students see the concepts through directed explorations and purposeful manipulation. Many of the instructional videos that accompany the text include Interactive Figures to teach key concepts. These figures are assignable in MyLab Math and encourage active learning, critical thinking, and conceptual understanding. The figures were created by Marc Renault (Shippensburg University), Steve Phelps (University of Cincinnati), Kevin Hopkins (Southwest Baptist University), and Tim Brzezinski (Southington High School, CT).



▼ **Instructional videos—**Hundreds of videos are available as learning aids within exercises and for self-study under the Video and Resource Library.



Other student resources include:

- **Student's Solutions Manual** The Student's Solutions Manual provides detailed worked-out solutions to the odd-numbered exercises in *Thomas' Calculus: Early Transcendentals,* 15th Edition in SI Units. Available in MyLab Math.
- **Companion Website**

  The companion Website, located at **www.pearsonglobaleditions.com**, offers an online appendix, three online chapters, historical biographies, and more.

Instructor Resources

Your course is unique. So, whether you'd like to build your own assignments, teach multiple sections, or set prerequisites, MyLab gives you the flexibility to easily create your course to fit your needs.

Pre-Built Assignments are designed to maximize students' performance. All assignments are *fully editable* to make your course your own.

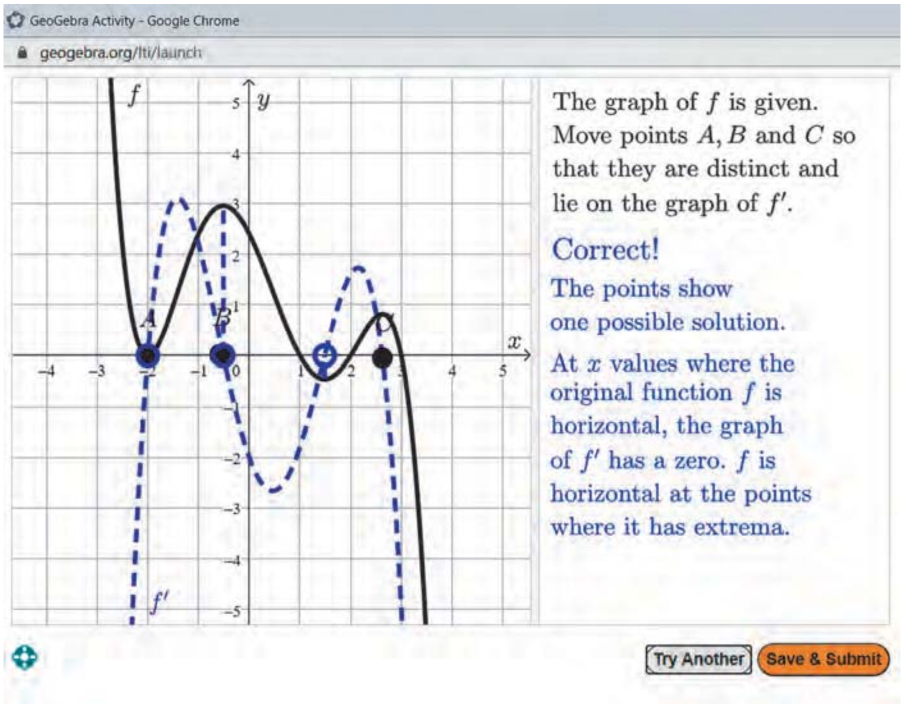
New! Video Assignments featuring short videos with corresponding MyLab Math exercises are available for each section of the textbook. These editable assignments are especially helpful for online or "flipped" classes, where some or all the learning takes place independently.

Enhanced Assignments—These section-level assignments have three unique properties:

1. They help keep skills fresh with *spaced practice* of previously learned concepts.
2. Learning aids are strategically turned off for some exercises to ensure students understand how to work the exercises independently.
3. They contain personalized prerequisite skills exercises for gaps identified in the chapter-level Skills Check Quiz.

MyLab Math Question Library is correlated to the exercises in the text, reflecting the authors' approach and learning style. They regenerate algorithmically to give students unlimited opportunity for practice and mastery. Below are a few exercise types available to assign:

▼ **New! GeoGebra Exercises** are gradable graphing and computational exercises that help students demonstrate their understanding. They enable students to interact directly with the graph in a manner that reflects how students would graph on paper.

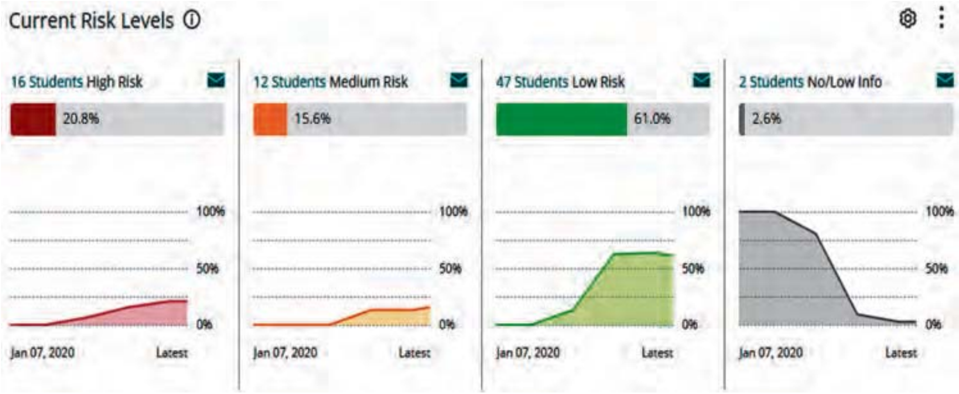


- **Nearly 100 More! Setup & Solve Exercises** require students to first describe how they will set up and approach the problem. This reinforces conceptual understanding of the process applied in approaching the problem, promotes long-term retention of the skill, and mirrors what students will be expected to do on a test. This new exercise type was widely praised by users of the 14th edition, so more were added to the 15th edition.
- **Conceptual Question Library** focuses on deeper, theoretical understanding of the key concepts in calculus. These questions were written by faculty at Cornell University under a National Science Foundation grant and are also assignable through Learning Catalytics.

Learning Catalytics—With Learning Catalytics, you'll hear from every student when it matters most. You pose a variety of questions in class (choosing from pre-loaded questions or your own) that help students recall ideas, apply concepts, and develop critical-thinking skills. Your students respond using their own smartphones, tablets, or laptops.

Performance Analytics enable instructors to see and analyze student performance across multiple courses. Based on their current course progress, individuals' performance is identified above, at, or below expectations through a variety of graphs and visualizations.

In Performance Analytics, instructors can email students individually or by group to provide feedback.



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Other instructor resources include:

- **Instructor's Solutions Manual—**The Instructor's Solutions Manual provides complete worked-out solutions for all exercises in *Thomas' Calculus: Early Transcendentals,* 15th Edition in SI Units. It can be downloaded from within MyLab Math or from Pearson's online catalog at www.pearsonglobaleditions.com.
- **PowerPoint Lecture Slides** feature editable lecture slides written and designed specifically for this text, including figures and examples.
- **TestGen** enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same questions or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download at www.pearsonglobaleditions.com.
- **Technology Manuals and Projects**

  Maple Manual and Projects
  Mathematica Manual and Projects
  TI-Graphing Calculator Manual

  These manuals and projects cover *Maple* 2021, *Mathematica* 12, and TI-84 Plus and TI-89. Each manual provides detailed guidance for integrating a specific software package or graphing calculator throughout the course, including syntax and commands. The projects include instructions and ready-made application files for *Maple* and *Mathematica*. Available to download within MyLab Math.

Learn more at https://www.pearsonmylabandmastering.com/global/

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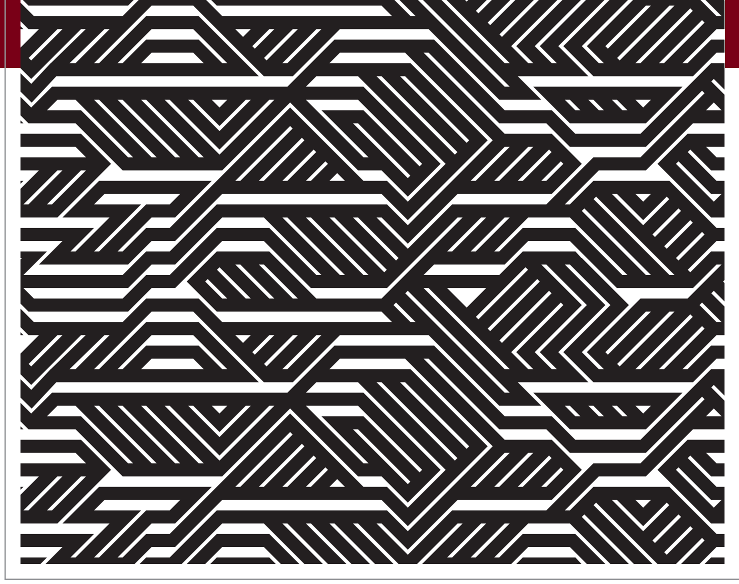
1

Functions



OVERVIEW In this chapter we review what functions are and how they are visualized as graphs, how they are combined and transformed, and ways they can be classified.

1.1 Functions and Their Graphs

Functions are a tool for describing the real world in mathematical terms. A function can be represented by an equation, a graph, a numerical table, or a verbal description; we will use all four representations throughout this text. This section reviews these ideas.

Functions; Domain and Range

The temperature at which water boils depends on the elevation above sea level. The interest paid on a cash investment depends on the length of time the investment is held. The area of a circle depends on the radius of the circle. The distance an object travels depends on the elapsed time.

In each case, the value of one variable quantity, say y, depends on the value of another variable quantity, which we often call x. We say that "y is a function of x" and write this symbolically as

$$y = f(x) \qquad (\text{"}y \text{ equals } f \text{ of } x\text{"}).$$

The symbol f represents the function, the letter x is the **independent variable** representing the input value to f, and y is the **dependent variable** or output value of f at x.

> DEFINITION A **function** f from a set D to a set Y is a rule that assigns a single value $f(x)$ in Y to each x in D.

A rule that assigns more than one value to an input x, such as the rule that assigns to a positive number both the positive and negative square roots of the number, does not describe a function.

The set D of all possible input values is called the **domain** of the function. The domain of f will sometimes be denoted by $D(f)$. The set of all output values $f(x)$ as x varies throughout D is called the **range** of the function. The range might not include every element in the set Y. The domain and range of a function can be any sets of objects, but often in calculus they are sets of real numbers interpreted as points of a coordinate line. (In Chapters 12–15, we will encounter functions for which the elements of the sets are points in the plane, or in space.)

Often a function is given by a formula that describes how to calculate the output value from the input variable. For instance, the equation $A = \pi r^2$ is a rule that calculates the area A of a circle from its radius r. When we define a function f with a formula $y = f(x)$ and the domain is not stated explicitly or restricted by context, the domain is assumed to be the largest set of real x-values for which the formula gives real y-values. This is called the **natural domain** of f. If we want to restrict the domain in some way, we must say so. The domain of $y = x^2$ is the entire set of real numbers. To restrict the domain of the function to, say, positive values of x, we would write "$y = x^2, x > 0$."

Changing the domain to which we apply a formula usually changes the range as well. The range of $y = x^2$ is $[0, \infty)$. The range of $y = x^2, x \geq 2$, is the set of all numbers obtained by squaring numbers greater than or equal to 2. In set notation (see Appendix A.1), the range is $\{x^2 | x \geq 2\}$ or $\{y | y \geq 4\}$ or $[4, \infty)$.

When the range of a function is a set of real numbers, the function is said to be **real-valued**. The domains and ranges of most real-valued functions we consider are intervals or combinations of intervals. Sometimes the range of a function is not easy to find.

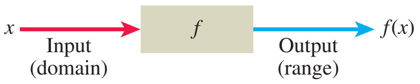


FIGURE 1.1 A diagram showing a function as a kind of machine.

A function f is like a machine that produces an output value $f(x)$ in its range whenever we feed it an input value x from its domain (Figure 1.1). The function keys on a calculator give an example of a function as a machine. For instance, whenever you enter a nonnegative number x and press the $\sqrt{x}$ key, the calculator gives an output value (the square root of x).

A function can also be pictured as an **arrow diagram** (Figure 1.2). Each arrow associates to an element of the domain D a single element in the set Y. In Figure 1.2, the arrows indicate that $f(a)$ is associated with a, $f(x)$ is associated with x, and so on. Notice that a function can have the same *output value* for two different input elements in the domain (as occurs with $f(a)$ in Figure 1.2), but each input element x is assigned a *single* output value $f(x)$.

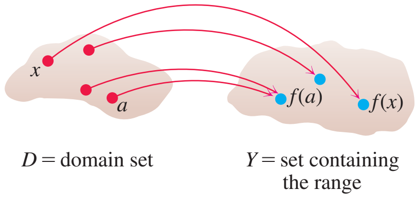


FIGURE 1.2 A function from a set D to a set Y assigns a unique element of Y to each element in D.

EXAMPLE 1 Verify the natural domains and associated ranges of some simple functions. The domains in each case are the values of x for which the formula makes sense.

Function	Domain (x)	Range (y)
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$

Solution The formula $y = x^2$ gives a real y-value for any real number x, so the domain is $(-\infty, \infty)$. The range of $y = x^2$ is $[0, \infty)$ because the square of any real number is nonnegative and every nonnegative number y is the square of its own square root: $y = \left(\sqrt{y}\right)^2$.

The formula $y = 1/x$ gives a real y-value for every x except $x = 0$. For consistency in the rules of arithmetic, *we cannot divide any number by zero*. The range of $y = 1/x$, the set of reciprocals of all nonzero real numbers, is the set of all nonzero real numbers, since $y = 1/(1/y)$. That is, for $y \neq 0$ the number $x = 1/y$ is the input that is assigned to the output value y.

The formula $y = \sqrt{x}$ gives a real y-value only if $x \geq 0$. The range of $y = \sqrt{x}$ is $[0, \infty)$ because every nonnegative number is some number's square root (namely, it is the square root of its own square).

In $y = \sqrt{4 - x}$, the quantity $4 - x$ cannot be negative. That is, $4 - x \geq 0$, or $x \leq 4$. The formula gives nonnegative real y-values for all $x \leq 4$. The range of $\sqrt{4 - x}$ is $[0, \infty)$, the set of all nonnegative numbers.

The formula $y = \sqrt{1 - x^2}$ gives a real y-value for every x in the closed interval from -1 to 1. Outside this domain, $1 - x^2$ is negative and its square root is not a real number. The values of $1 - x^2$ vary from 0 to 1 on the given domain, and the square roots of these values do the same. The range of $\sqrt{1 - x^2}$ is $[0, 1]$. ■

Graphs of Functions

If f is a function with domain D, its **graph** consists of the points in the Cartesian plane whose coordinates are the input-output pairs for f. In set notation, the graph is

$$\{(x, f(x)) \mid x \in D\}.$$

The graph of the function $f(x) = x + 2$ is the set of points with coordinates (x, y) for which $y = x + 2$. Its graph is the straight line sketched in Figure 1.3.

The graph of a function f is a useful picture of its behavior. If (x, y) is a point on the graph, then $y = f(x)$ is the height of the graph above (or below) the point x. The height may be positive or negative, depending on the sign of $f(x)$ (Figure 1.4).

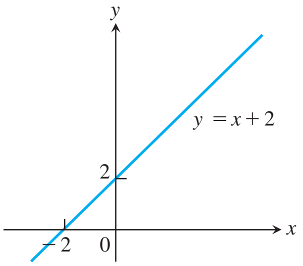


FIGURE 1.3 The graph of $f(x) = x + 2$ is the set of points (x, y) for which y has the value $x + 2$.

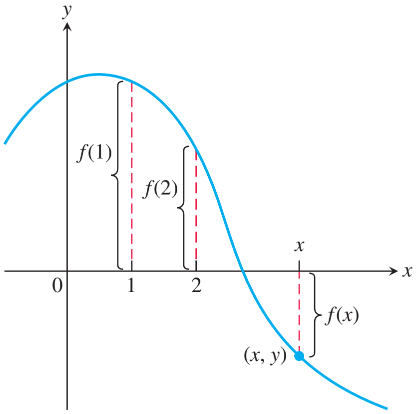


FIGURE 1.4 If (x, y) lies on the graph of f, then the value $y = f(x)$ is the height of the graph above the point x (or below x if $f(x)$ is negative).

EXAMPLE 2 Graph the function $y = x^2$ over the interval $[-2, 2]$.

Solution Make a table of xy-pairs that satisfy the equation $y = x^2$. Plot the points (x, y) whose coordinates appear in the table, and draw a *smooth* curve (labeled with its equation) through the plotted points (see Figure 1.5).

x	$y = x^2$
−2	4
−1	1
0	0
1	1
$\frac{3}{2}$	$\frac{9}{4}$
2	4

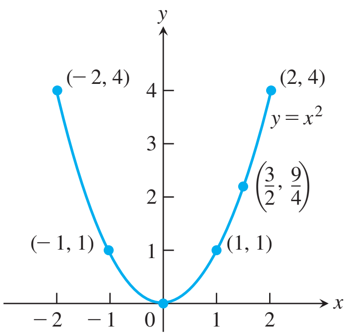
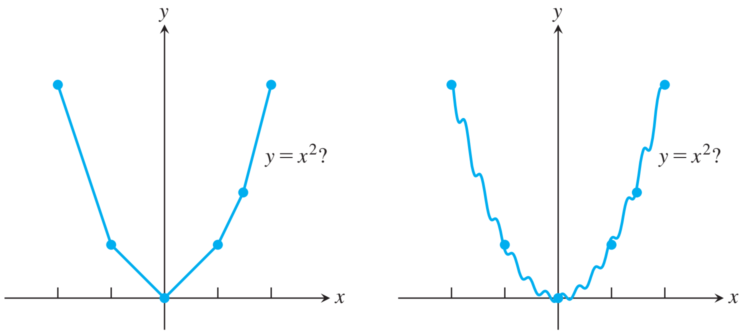


FIGURE 1.5 Graph of the function in Example 2.

How do we know that the graph of $y = x^2$ doesn't look like one of these curves?



To find out, we could plot more points. But how would we then connect *them*? The basic question still remains: How do we know for sure what the graph looks like between the points we plot? Calculus answers this question, as we will see in Chapter 4. Meanwhile, we will have to settle for plotting points and connecting them as best we can.

Time	Pressure
0.00091	–0.080
0.00108	0.200
0.00125	0.480
0.00144	0.693
0.00162	0.816
0.00180	0.844
0.00198	0.771
0.00216	0.603
0.00234	0.368
0.00253	0.099
0.00271	–0.141
0.00289	–0.309
0.00307	–0.348
0.00325	–0.248
0.00344	–0.041
0.00362	0.217
0.00379	0.480
0.00398	0.681
0.00416	0.810
0.00435	0.827
0.00453	0.749
0.00471	0.581
0.00489	0.346
0.00507	0.077
0.00525	–0.164
0.00543	–0.320
0.00562	–0.354
0.00579	–0.248
0.00598	–0.035

Representing a Function Numerically

A function may be represented algebraically by a formula and visually by a graph (Example 2). Another way to represent a function is **numerically**, through a table of values. From an appropriate table of values, a graph of the function can be obtained using the method illustrated in Example 2, possibly with the aid of a computer. The graph consisting of only the points in the table is called a **scatterplot**.

EXAMPLE 3 Musical notes are pressure waves in the air. The data associated with Figure 1.6 give recorded pressure displacement versus time in seconds of a musical note produced by a tuning fork. The table provides a representation of the pressure function (in micropascals) over time. If we first make a scatterplot and then draw a smooth curve that approximates the data points (t, p) from the table, we obtain the graph shown in the figure.

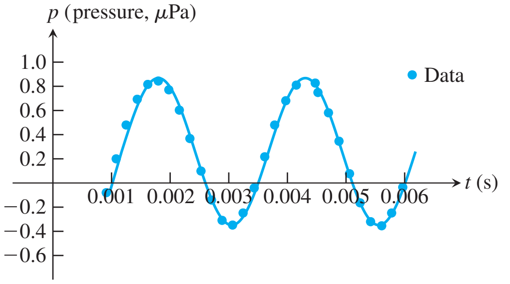


FIGURE 1.6 A smooth curve approximating the plotted points gives a graph of the pressure function represented by the accompanying tabled data (Example 3).

The Vertical Line Test for a Function

Not every curve in the coordinate plane can be the graph of a function. A function f can have only one value $f(x)$ for each x in its domain, so *no vertical line* can intersect the graph of a function at more than one point. If a is in the domain of the function f, then the vertical line $x = a$ will intersect the graph of f at the single point $(a, f(a))$.

A circle cannot be the graph of a function, since some vertical lines intersect the circle twice. The circle graphed in Figure 1.7a, however, contains the graphs of two functions of x, namely the upper semicircle defined by the function $f(x) = \sqrt{1 - x^2}$ and the lower semicircle defined by the function $g(x) = -\sqrt{1 - x^2}$ (Figures 1.7b and 1.7c).

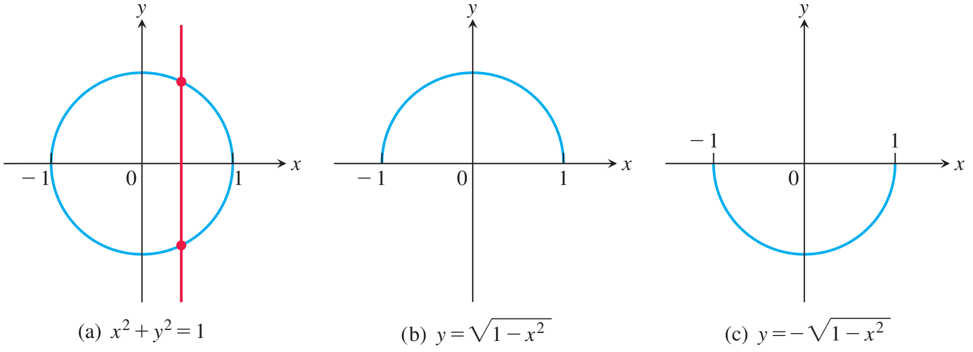


FIGURE 1.7 (a) The circle is not the graph of a function; it fails the vertical line test. (b) The upper semicircle is the graph of the function $f(x) = \sqrt{1 - x^2}$. (c) The lower semicircle is the graph of the function $g(x) = -\sqrt{1 - x^2}$.

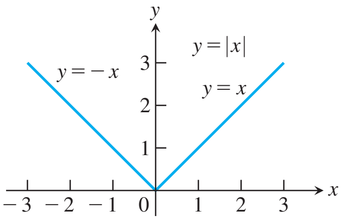


FIGURE 1.8 The absolute value function has domain $(-\infty, \infty)$ and range $[0, \infty)$.

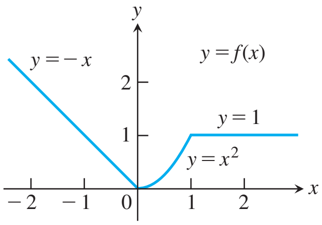


FIGURE 1.9 To graph the function $y = f(x)$ shown here, we apply different formulas to different parts of its domain (Example 4).

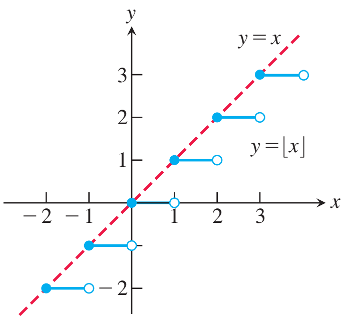


FIGURE 1.10 The graph of the greatest integer function $y = \lfloor x \rfloor$ lies on or below the line $y = x$, so it provides an integer floor for x (Example 5).

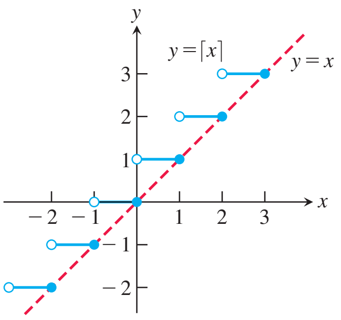


FIGURE 1.11 The graph of the least integer function $y = \lceil x \rceil$ lies on or above the line $y = x$, so it provides an integer ceiling for x (Example 6).

Piecewise-Defined Functions

Sometimes a function is described in pieces by using different formulas on different parts of its domain. One example is the **absolute value function**

$$|x| = \begin{cases} x, & x \geq 0 \quad \text{First formula} \\ -x, & x < 0 \quad \text{Second formula} \end{cases}$$

whose graph is given in Figure 1.8. The right-hand side of the equation means that the function equals x if $x \geq 0$, and equals $-x$ if $x < 0$. Piecewise-defined functions often arise when real-world data are modeled. Here are some other examples.

EXAMPLE 4 The function

$$f(x) = \begin{cases} -x, & x < 0 \quad \text{First formula} \\ x^2, & 0 \leq x \leq 1 \quad \text{Second formula} \\ 1, & x > 1 \quad \text{Third formula} \end{cases}$$

is defined on the entire real line but has values given by different formulas, depending on the position of x. The values of f are given by $y = -x$ when $x < 0$, $y = x^2$ when $0 \leq x \leq 1$, and $y = 1$ when $x > 1$. The function, however, is *just one function* whose domain is the entire set of real numbers (Figure 1.9). ■

EXAMPLE 5 The function whose value at any number x is the *greatest integer less than or equal to* x is called the **greatest integer function** or the **integer floor function**. It is denoted $\lfloor x \rfloor$. Figure 1.10 shows the graph. Observe that

$$\lfloor 2.4 \rfloor = 2, \quad \lfloor 1.9 \rfloor = 1, \quad \lfloor 0 \rfloor = 0, \quad \lfloor -1.2 \rfloor = -2,$$
$$\lfloor 2 \rfloor = 2, \quad \lfloor 0.2 \rfloor = 0, \quad \lfloor -0.3 \rfloor = -1, \quad \lfloor -2 \rfloor = -2.$$ ■

EXAMPLE 6 The function whose value at any number x is the *smallest integer greater than or equal to* x is called the **least integer function** or the **integer ceiling function**. It is denoted $\lceil x \rceil$. Figure 1.11 shows the graph. For positive values of x, this function might represent, for example, the cost of parking x hours in a parking lot that charges \$1 for each hour or part of an hour. ■

Increasing and Decreasing Functions

If the graph of a function climbs or rises as you move from left to right, we say that the function is *increasing*. If the graph descends or falls as you move from left to right, the function is *decreasing*.

DEFINITIONS Let f be a function defined on an interval I and let x_1 and x_2 be two distinct points in I.

1. If $f(x_2) > f(x_1)$ whenever $x_1 < x_2$, then f is said to be **increasing** on I.
2. If $f(x_2) < f(x_1)$ whenever $x_1 < x_2$, then f is said to be **decreasing** on I.

It is important to realize that the definitions of increasing and decreasing functions must be satisfied for *every* pair of points x_1 and x_2 in I with $x_1 < x_2$. Because we use the inequality $<$ to compare the function values, instead of $\leq$, it is sometimes said that f is *strictly* increasing or decreasing on I. The interval I may be finite (also called bounded) or infinite (unbounded).

EXAMPLE 7 The function graphed in Figure 1.9 is decreasing on $(-\infty, 0)$ and increasing on $(0, 1)$. The function is neither increasing nor decreasing on the interval $(1, \infty)$ because the function is constant on that interval, and hence the strict inequalities in the definition of increasing or decreasing are not satisfied on $(1, \infty)$. ■

Even Functions and Odd Functions: Symmetry

The graphs of *even* and *odd* functions have special symmetry properties.

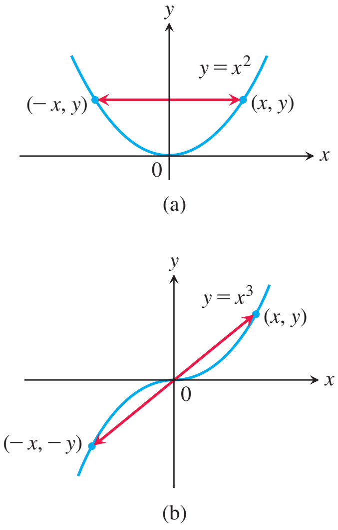


FIGURE 1.12 (a) The graph of $y = x^2$ (an even function) is symmetric about the y-axis. (b) The graph of $y = x^3$ (an odd function) is symmetric about the origin.

DEFINITIONS A function $y = f(x)$ is an

even function of x if $f(-x) = f(x)$,

odd function of x if $f(-x) = -f(x)$,

for every x in the function's domain.

The names *even* and *odd* come from powers of x. If y is an even power of x, as in $y = x^2$ or $y = x^4$, it is an even function of x because $(-x)^2 = x^2$ and $(-x)^4 = x^4$. If y is an odd power of x, as in $y = x$ or $y = x^3$, it is an odd function of x because $(-x)^1 = -x$ and $(-x)^3 = -x^3$.

The graph of an even function is **symmetric about the y-axis**. Since $f(-x) = f(x)$, a point (x, y) lies on the graph if and only if the point $(-x, y)$ lies on the graph (Figure 1.12a). A reflection across the y-axis leaves the graph unchanged.

The graph of an odd function is **symmetric about the origin**. Since $f(-x) = -f(x)$, a point (x, y) lies on the graph if and only if the point $(-x, -y)$ lies on the graph (Figure 1.12b). Equivalently, a graph is symmetric about the origin if a rotation of 180° about the origin leaves the graph unchanged.

Notice that each of these definitions requires that both x and $-x$ be in the domain of f.

EXAMPLE 8 Here are several functions illustrating the definitions.

$f(x) = x^2$ — Even function: $(-x)^2 = x^2$ for all x; symmetry about y-axis. So $f(-3) = 9 = f(3)$. Changing the sign of x does not change the value of an even function.

$f(x) = x^2 + 1$ — Even function: $(-x)^2 + 1 = x^2 + 1$ for all x; symmetry about y-axis (Figure 1.13a).

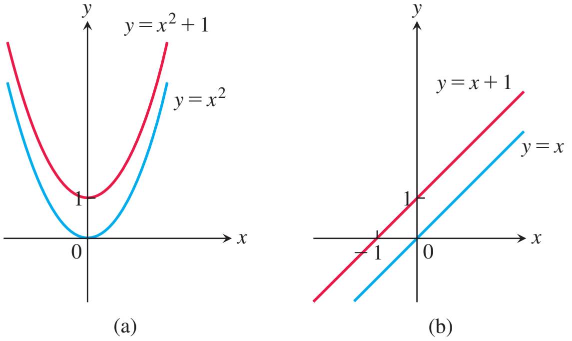


FIGURE 1.13 (a) When we add the constant term 1 to the function $y = x^2$, the resulting function $y = x^2 + 1$ is still even and its graph is still symmetric about the y-axis. (b) When we add the constant term 1 to the function $y = x$, the resulting function $y = x + 1$ is no longer odd, since the symmetry about the origin is lost. The function $y = x + 1$ is also not even (Example 8).

$f(x) = x$ — Odd function: $(-x) = -x$ for all x; symmetry about the origin. So $f(-3) = -3$ while $f(3) = 3$. Changing the sign of x changes the sign of the value of an odd function.

$f(x) = x + 1$ — Not odd: $f(-x) = -x + 1$, but $-f(x) = -x - 1$. The two are not equal.

Not even: $(-x) + 1 \neq x + 1$ for all $x \neq 0$ (Figure 1.13b).

Common Functions

A variety of important types of functions are frequently encountered in calculus.

Linear Functions A function of the form $f(x) = mx + b$, where m and b are fixed constants, is called a **linear function**. Figure 1.14a shows an array of lines $f(x) = mx$. Each of these has $b = 0$, so these lines pass through the origin. The function $f(x) = x$, where $m = 1$ and $b = 0$, is called the **identity function**. Constant functions result when the slope is $m = 0$ (Figure 1.14b).

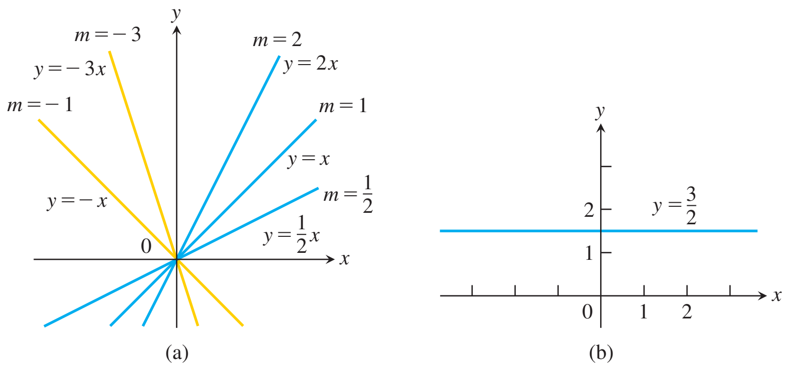


FIGURE 1.14 (a) Lines through the origin with slope m. (b) A constant function with slope $m = 0$.

> **DEFINITION** Two variables y and x are **proportional** (to one another) if one is always a constant multiple of the other—that is, if $y = kx$ for some nonzero constant k.

If the variable y is proportional to the reciprocal $1/x$, then sometimes it is said that y is **inversely proportional** to x (because $1/x$ is the multiplicative inverse of x).

Power Functions A function $f(x) = x^a$, where a is a constant, is called a **power function**. There are several important cases to consider.

(a) $f(x) = x^a$ with $a = n$, a positive integer.

The graphs of $f(x) = x^n$, for $n = 1, 2, 3, 4, 5$, are displayed in Figure 1.15. These functions are defined for all real values of x. Notice that as the power n gets larger, the curves tend to flatten toward the x-axis on the interval $(-1, 1)$ and to rise more steeply for $|x| > 1$. Each curve passes through the point $(1, 1)$ and through the origin. The graphs of functions with even powers are symmetric about the y-axis; those with odd powers are symmetric about the origin. The even-powered functions are decreasing on the interval $(-\infty, 0]$ and increasing on $[0, \infty)$; the odd-powered functions are increasing over the entire real line $(-\infty, \infty)$.

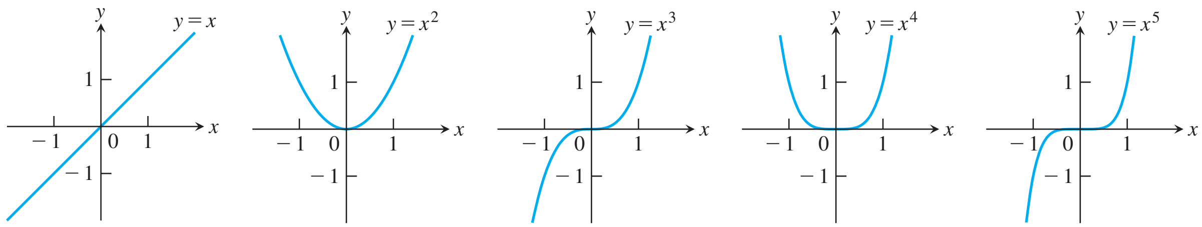


FIGURE 1.15 Graphs of $f(x) = x^n, n = 1, 2, 3, 4, 5$, defined for $-\infty < x < \infty$.

(b) $f(x) = x^a$ with $a = -1$ or $a = -2$.

The graphs of the functions $f(x) = x^{-1} = 1/x$ and $f(x) = x^{-2} = 1/x^2$ are shown in Figure 1.16. Both functions are defined for all $x \neq 0$ (you can never divide by zero). The graph of $y = 1/x$ is the hyperbola $xy = 1$, which approaches the coordinate axes far from the origin. The graph of $y = 1/x^2$ also approaches the coordinate axes. The graph of the function $f(x) = 1/x$ is symmetric about the origin; this function is decreasing on the intervals $(-\infty, 0)$ and $(0, \infty)$. The graph of the function $f(x) = 1/x^2$ is symmetric about the y-axis; this function is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$.

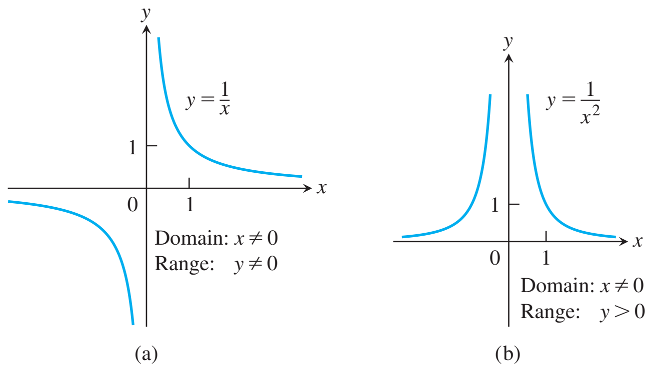


FIGURE 1.16 Graphs of the power functions $f(x) = x^a$. (a) $a = -1$. (b) $a = -2$.

(c) $f(x) = x^a$ with $a = \frac{1}{2}, \frac{1}{3}, \frac{3}{2}$, or $\frac{2}{3}$.

The functions $f(x) = x^{1/2} = \sqrt{x}$ and $f(x) = x^{1/3} = \sqrt[3]{x}$ are the **square root** and **cube root** functions, respectively. The domain of the square root function is $[0, \infty)$, but the cube root function is defined for all real x. Their graphs are displayed in Figure 1.17, along with the graphs of $y = x^{3/2}$ and $y = x^{2/3}$. (Recall that $x^{3/2} = (x^{1/2})^3$ and $x^{2/3} = (x^{1/3})^2$.)

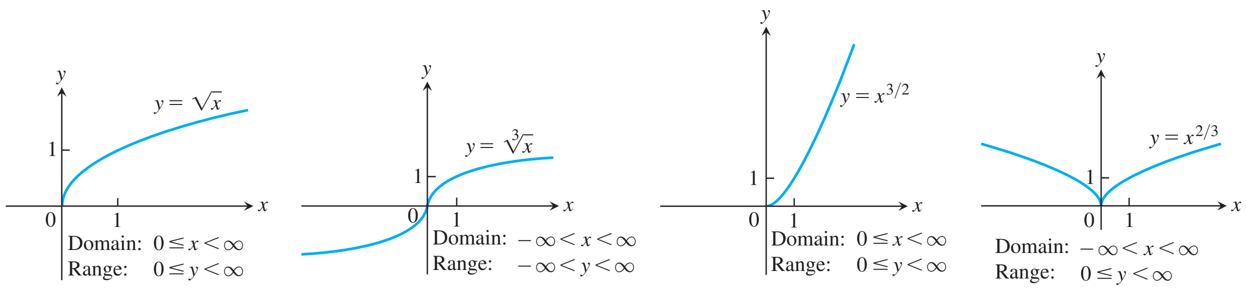


FIGURE 1.17 Graphs of the power functions $f(x) = x^a$ for $a = \frac{1}{2}, \frac{1}{3}, \frac{3}{2}$, and $\frac{2}{3}$.

Polynomials A function p is a **polynomial** if

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where n is a nonnegative integer and the numbers $a_0, a_1, a_2, \ldots, a_n$ are real constants (called the **coefficients** of the polynomial). All polynomials have domain $(-\infty, \infty)$. If the leading coefficient $a_n \neq 0$, then n is called the **degree** of the polynomial. Linear functions with $m \neq 0$ are polynomials of degree 1. Polynomials of degree 2, usually written as $p(x) = ax^2 + bx + c$, are called **quadratic functions**. Likewise, **cubic functions** are polynomials $p(x) = ax^3 + bx^2 + cx + d$ of degree 3. Figure 1.18 shows the graphs of three polynomials. Techniques to graph polynomials are studied in Chapter 4.

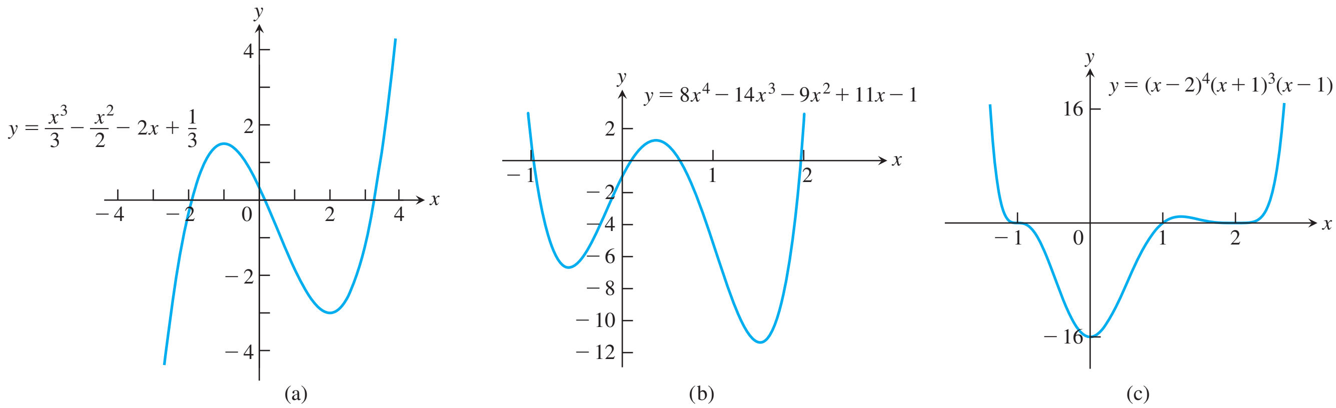


FIGURE 1.18 Graphs of three polynomial functions.

Rational Functions A **rational function** is a quotient or ratio $f(x) = p(x)/q(x)$, where p and q are polynomials. The domain of a rational function is the set of all real x for which $q(x) \neq 0$. The graphs of three rational functions are shown in Figure 1.19.

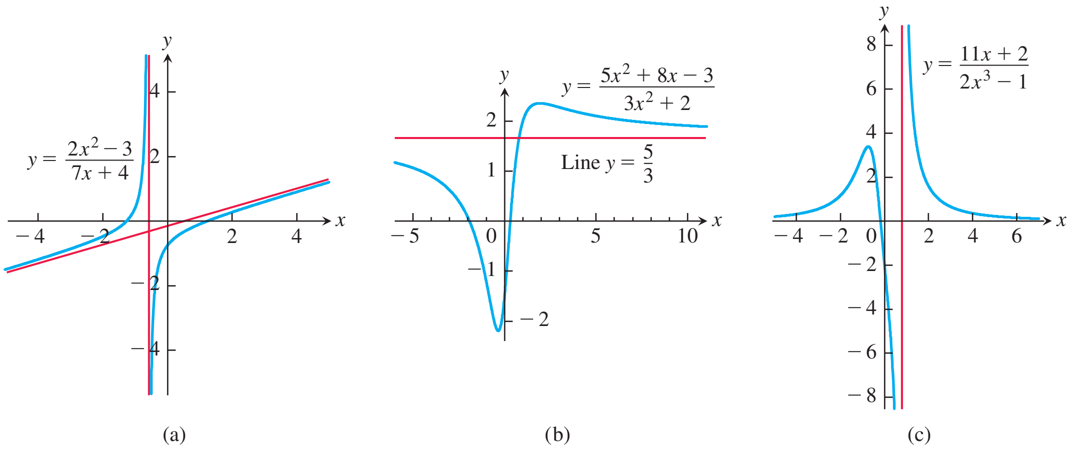


FIGURE 1.19 Graphs of three rational functions. The straight red lines approached by the graphs are called *asymptotes* and are not part of the graphs. We discuss asymptotes in Section 2.5.

Algebraic Functions Any function constructed from polynomials using algebraic operations (addition, subtraction, multiplication, division, and taking roots) lies within the class of **algebraic functions**. All rational functions are algebraic, but also included are more complicated functions (such as those satisfying an equation like $y^3 - 9xy + x^3 = 0$, studied in Section 3.7). Figure 1.20 displays the graphs of three algebraic functions.

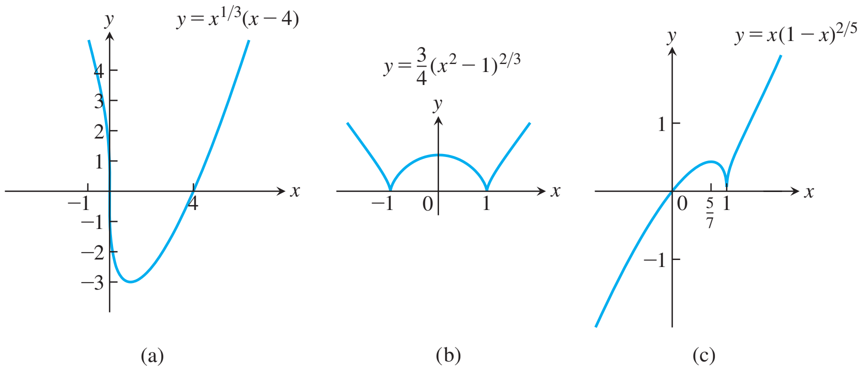


FIGURE 1.20 Graphs of three algebraic functions.

Trigonometric Functions The six basic trigonometric functions are reviewed in Section 1.3. The graphs of the sine and cosine functions are shown in Figure 1.21.

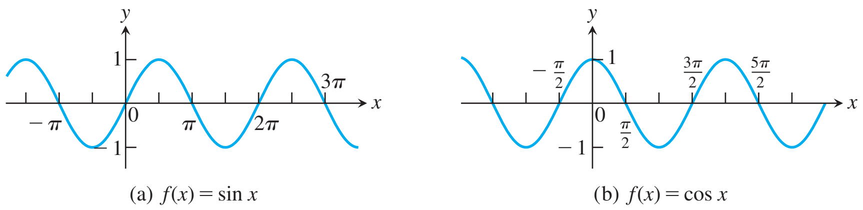


FIGURE 1.21 Graphs of the sine and cosine functions.

Exponential Functions A function of the form $f(x) = a^x$, where $a > 0$ and $a \neq 1$, is called an **exponential function** (with base a). All exponential functions have domain $(-\infty, \infty)$ and range $(0, \infty)$, so an exponential function never assumes the value 0. We discuss exponential functions in Section 1.4. The graphs of some exponential functions are shown in Figure 1.22.

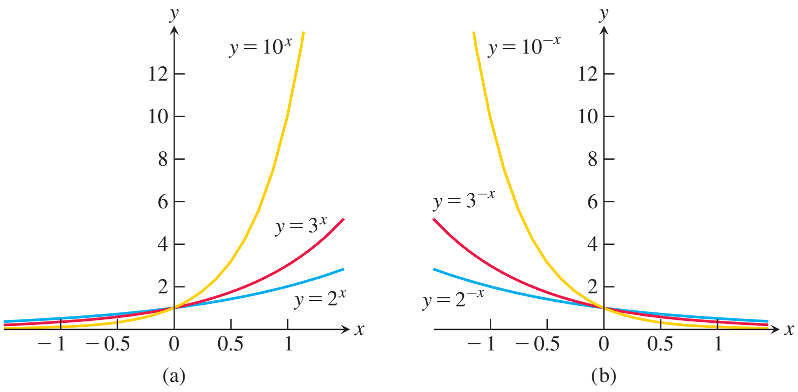


FIGURE 1.22 Graphs of exponential functions.