



# Tenth Edition

# PRECALCULUS GRAPHICAL, NUMERICAL, ALGEBRAIC

Demana • Waits • Foley • Kennedy • Bock



# Precalculus Graphical, Numerical, Algebraic

Tenth Edition Global Edition

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# FOREWORD

We are proud of the fact that earlier editions of *Precalculus: Graphical, Numerical, Algebraic* were among the first to recognize the potential of hand-held graphers for helping students understand function behavior. The power of visualization eventually transformed the teaching and learning of calculus at the college level and in the AP<sup>®</sup> program, then led to reforms in the high school curriculum articulated in the NCTM *Principles and Standards for School Mathematics* and more recently in the Common Core State Standards. All along the way, this text has kept current with the best practices while continuing to pioneer new ideas in exploration and pedagogy that enhance student learning (for example, the study of function behavior based on the Twelve Basic Functions, an idea that has gained widespread acceptance in the text world).

For those students continuing to a calculus course, this precalculus text concludes with a chapter that prepares students for the two central themes of calculus: instantaneous rate of change and continuous accumulation. This intuitively appealing preview of calculus is both more useful and more reasonable than the traditional, unmotivated foray into the computation of limits, and it is more in keeping with the stated goals and objectives of the AP courses and their emphasis on depth of knowledge.

Recognizing that precalculus is a capstone course for many students, we include *quantitative literacy* topics such as probability, statistics, and the mathematics of finance and integrate the use of data and modeling throughout the text. Our goal is to provide students with the critical-thinking skills and mathematical know-how needed to succeed in college, career, or any endeavor.

Continuing in the spirit of the nine earlier editions, we have integrated graphing technology throughout the course, not as an additional topic but as an essential tool for both mathematical discovery and effective problem solving. Graphing technology enables students to study a full catalog of basic functions at the beginning of the course, thereby giving them insights into function properties that are not seen in many texts until later chapters. By connecting the algebra of functions to the visualization of their graphs, we are even able to introduce students to parametric equations, piecewise-defined functions, limit notation, and an intuitive understanding of continuity as early as Chapter 1. However, the advances in technology and increased familiarity with calculators have blurred some of the distinctions between solving problems and supporting solutions that we had once assumed to be apparent. Therefore, we ask that some exercises be solved without calculators. (See the Technology and Exercises section of the Preface.)

Once students are comfortable with the language of functions, the text guides them through a more traditional exploration of twelve basic functions and their algebraic properties, always reinforcing the connections among their algebraic, graphical, and numerical representations. This text uses a consistent approach to modeling, emphasizing the use of particular types of functions to model behavior in the real world. Modeling is a fundamental aspect of our problem-solving process that is introduced in Section 1.1 and used throughout the text. The text has a wealth of data and range of applications to illustrate how mathematics and statistics connect to every facet of modern life. Each chapter, 1–11, concludes with a modeling project to reinforce and extend students' ability to solve modeling problems.

This text has faithfully incorporated not only the teaching strategies that have made *Calculus: Graphical, Numerical, Algebraic* so popular, but also some of the strategies from the popular Pearson high school algebra series, and thus has produced a seamless pedagogical transition from prealgebra through calculus for students. Although this

book can certainly be appreciated on its own merits, teachers who seek coherence and vertical alignment in their mathematics sequence might consider this pedagogical approach to be an additional asset of *Precalculus: Graphical, Numerical, Algebraic*.

This text is written to address current and emerging state curriculum standards. In particular, we embrace NCTM's *Focus in High School Mathematics: Reasoning and Sense Making* and its emphasis on the importance of helping students to make sense of mathematics and to reason using mathematics. The NCTM's *Principles and Standards for School Mathematics* identified five "Process Standards" that should be fundamental in mathematics education. The first of these standards was Problem Solving. Since then, the emphasis on problem solving has continued to grow, to the point that it is now integral to the instructional process in many mathematics classrooms. When the Common Core State Standards for Mathematics detailed eight "Standards for Mathematical Practice" that should be fundamental in mathematics education, again the first of these addressed problem solving. Individual states have also released their own standards over the years, and problem solving is invariably front and center as a fundamental objective. Problem solving, reasoning, sense making, and the related processes and practices of mathematics are central to the approach we use in *Precalculus: Graphical, Numerical, Algebraic.* 

We embrace the growing importance and wide applicability of Statistics. Because Statistics is increasingly used in college coursework, the workplace, and everyday life, we include a full chapter on Statistics to help students see that statistical analysis is an investigative process that turns loosely formed ideas into scientific studies. Our five sections on data analysis, probability, and statistical literacy are aligned with the *GAISE* Report published by the American Statistical Association, the College Board's AP<sup>®</sup> Statistics curriculum, and the Common Core State Standards. Chapter 10 is not intended as a course in statistics but rather as an introduction to set the stage for possible further study.

#### **Dedication**

We dedicate this text to the memory of our eminent colleague, dear friend, and inspirational coauthor **Bert K. Waits** (1940–2014).

With his passing, the mathematics community lost a uniquely talented leader. May he rest in peace, and may the power of visualization, which he passionately promoted, live on!

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#### David E. Bock

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Mr. Bock coauthored the AP Statistics textbook *Stats: Modeling the World*, the non-AP text *Stats in Your World*, Barron's *AP Calculus* review book, and Barron's *AP Calculus Flash Cards*.

#### **Our Approach**

#### The Rule of Four—A Balanced Approach

A principal feature of this text is the balance among the algebraic, numerical, graphical, and verbal methods of representing problems: the rule of four. For instance, we obtain solutions algebraically when that is the most appropriate technique to use, and we obtain solutions graphically or numerically when algebra is difficult to use. We urge students to solve problems by one method and then to support or confirm their solutions by using another method. We believe that students must learn the value of each of these methods or representations and must learn to choose the one most appropriate for solving the particular problem under consideration. This approach reinforces the idea that to understand a problem fully, students need to understand it algebraically as well as graphically and numerically.

#### **Problem-Solving Approach**

Systematic problem solving is emphasized in the examples throughout the text, using the following variation of Polya's problem-solving process that emphasizes modeling with mathematics:

- understand the problem,
- develop a mathematical model for the problem,
- solve the mathematical model and support or confirm the solution, and
- *interpret* the solution within the problem setting.

Students are encouraged to use this process throughout the text.

#### **Twelve Basic Functions**

Twelve basic functions are emphasized throughout the text as a major theme and focus. These functions are

- The Identity Function The Natural Logarithm Function
- The Squaring Function
   The Sine Function
- The Cubing Function
   The Cosine Function
- The Reciprocal Function
   The Absolute Value Function
- The Square Root Function The Greatest Integer Function
- The Exponential Function
   • The Logistic Function

One of the most distinctive features of this text is that it introduces students to the full vocabulary of functions early in the course. Students meet the twelve basic functions graphically in Chapter 1 and are able to compare and contrast them as they learn about concepts like domain, range, symmetry, continuity, end behavior, asymptotes, extrema, and even periodicity—concepts that are difficult to appreciate when the only examples a teacher can refer to are polynomials. With this text, students are able to characterize functions by their behavior within the first month of classes. Once students have a comfortable understanding of functions in general, the rest of the course consists of studying the various types of functions in greater depth, particularly with respect to their algebraic properties and modeling applications.

These functions are used to develop the fundamental analytic skills that are needed in calculus and advanced mathematics courses. A complete gallery of basic functions is included in Appendix C.

#### Applications, Data, and Modeling

The majority of the applications in the text are based on real data from cited sources, and their presentations are self-contained. As they work through the applications, students are exposed to functions as mechanisms for modeling real-life problems. They learn to analyze and model data, represent data graphically, interpret graphs, and fit curves. Additionally, the tabular representation of data presented in this text highlights the concept that a function is a correspondence between numerical variables. This helps students build the connection between numerical quantities and graphs and recognize the importance of a full graphical, numerical, and algebraic understanding of a problem. For a complete listing of applications, please see the Applications Index on page 947.

#### **Technology and Exercises**

The authors of this text have encouraged the use of technology in mathematics education for more than three decades. Our approach to problem solving (pages 92–93) distinguishes between **solving** the problem and **supporting** or **confirming** the solution, and emphasizes how technology figures in each of these processes.

We have come to realize, however, that advances in technology and increased familiarity with calculators have gradually blurred some of the distinctions between solving and supporting that we had once assumed to be apparent. We do not want to retreat in any way from our support of modern technology, but we now provide specific guidance about the intent of the various exercises in our text.

Therefore, as a service to teachers and students alike, exercises in this text that **should be solved without calculators** are identified with gray ovals around the exercise numbers. These usually are exercises that demonstrate how various functions behave algebraically or how algebraic representations reflect graphical behavior and vice versa. Application problems usually have no restrictions, in keeping with our emphasis on **modeling** and on bringing **all representations** to bear when confronting real-world problems.

Incidentally, we continue to encourage the use of calculators to **support** answers graphically or numerically after the problems have been solved with pencil and paper. Any time students can make connections among the graphical, analytical, and numerical representations, they are doing good mathematics.

As a final note, we will freely admit that different teachers use our text in different ways, and some will probably override our no-calculator recommendations to fit with their pedagogical strategies. In the end, the teachers know what is best for their students, and we are just here to help.

#### **Content Changes to This Edition**

Although the table of contents is essentially the same, this edition includes numerous substantial changes. About 5% of the examples have been replaced; another 5% have new data or new contexts. Additionally, 15–20% of the examples have been replaced and or clarified in some way. As for the exercises, again, about 10% have been replaced and another 5% have new data or new contexts. Plus, 5–10% of the exercises have been enhanced or clarified in some way. In particular, to keep the applications of mathematics relevant to our students, we have included the most current data available to us at the time of publication. As an example, look at the Chapter Opener problem on page 166. Not only does this include current data but also an entirely new twist: piecewise modeling.

Several other changes have been made as well. We have updated many of the student and teacher notes. We have updated calculator screenshots to conform to the enhanced capabilities of modern graphing calculators. We have updated and renamed the capstone projects for Chapters 1–11 as Modeling Projects to reflect that they can be used as a bridge to the open-ended modeling recommended in the *GAIMME* report, published in 2016 by the Consortium for Mathematics and Its Applications (COMAP) and the Society for Industrial and Applied Mathematics (SIAM).

#### **Features**

**Chapter Openers** include a general description of an application that can be solved with the concepts learned in the chapter. The application is revisited later in the chapter via a specific problem that is solved.

A **Chapter Overview** begins each chapter to give students a sense of what they are going to learn. This overview provides a roadmap of the chapter and also indicates how the topics in the chapter are connected under one big idea. It is always helpful to remember that mathematics isn't modular, but interconnected, and that the skills and concepts learned throughout the course build on one another to help students understand more complicated processes and relationships. Similarly, the **What you'll learn about ...** and why feature presents the big ideas in each section and explains their purpose.

Throughout the text, **Vocabulary** is highlighted in yellow for easy reference. Additionally, **Properties**, **Definitions**, and **Theorems** are boxed in purple, and **Procedures** in green, so that they can be easily found. The **Web/Real Data** icon marks the examples and exercises that use data cited from authentic sources.

Each example ends with a suggestion to **Now try** a related exercise. Working the suggested exercise is an easy way for students to check their comprehension of the material while reading each section. Alternatives are provided for these examples in the PowerPoint Slides.

**Explorations** appear throughout the text and provide students with the perfect opportunity to become active learners and to discover mathematics on their own. This will help hone critical-thinking and problem-solving skills. Some are technology-based; others involve exploring mathematical ideas and connections.

**Margin Notes** on various topics appear throughout the text. Some of these offer practical advice on using a grapher to obtain the best, most accurate results. Other notes include historical information, give hints about examples, or provide insight to help students avoid common pitfalls and errors.

The **Looking Ahead to Calculus** icon is found throughout the text next to many examples and topics to point out concepts that students will encounter again in calculus. Ideas that foreshadow calculus, such as limits, maximum and minimum, asymptotes, and continuity, are highlighted. Some calculus notation and language are introduced in the early chapters and used throughout the text to establish familiarity.

The review material at the end of each chapter consists of sections dedicated to helping students review the chapter concepts. **Key Ideas** are broken into parts: Properties, Theorems, and Formulas; Procedures; and Gallery of Functions. The **Review Exercises** represent the full range of exercises covered in the chapter and give additional practice with the ideas developed in the chapter. The exercises with red numbers indicate problems that would make up a good chapter test. A **Modeling Project** concludes each chapter and requires students to analyze data. It can be assigned as either individual or group work. Each project expands upon concepts and ideas taught in the chapter and engages students in modeling with mathematics.

#### **Exercise Sets**

Each exercise set begins with a **Quick Review** to help students review skills needed in the exercise set and refers them to other sections they can go to for help. Some exercises are designed to be solved *without* a *calculator*; the numbers of these exercises are printed within a gray oval. Students are urged to **support** the answers to these (and all) exercises graphically or numerically, but only after they have solved them with pencil and paper.

There are over 6000 exercises, including 720 Quick Review Exercises. The section exercises have been carefully graded from routine to challenging. The following types of skills are tested in each exercise set:

- · Algebraic understanding and procedures
- · Applications of mathematics
- · Connecting algebra to geometry
- · Interpretation of graphs
- · Graphical and numerical representations of functions
- Data analysis

The exercise sets include distinctive kinds of thought-provoking exercises:

- Standardized Test Questions include two true-false problems with justifications and four multiple-choice questions.
- **Explorations** are opportunities for students to discover mathematics on their own or in groups. These exercises often require the use of critical thinking to explore the ideas involved.
- Writing to Learn exercises give students practice at communicating about mathematics and opportunities to demonstrate their understanding of important ideas.
- **Group Activity** exercises ask students to work collaboratively to solve problems while interacting with a few of their classmates.
- Extending the Ideas exercises go beyond what is presented in the text. These exercises are challenging extensions of the material in the text.

This variety of exercises provides sufficient flexibility to emphasize the skills and concepts most needed for each student or class.

#### **Technology Resources**

The following supplements are available for purchase:

# MyLab Math Online Course (optional, for purchase only)—access code required

MyLab Math delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And it comes from a **trusted partner** with educational expertise and an eye on the future. To learn more about how MyLab Math combines proven learning applications with powerful assessment, visit **https://mlm.pearson.com/global/** or contact your Pearson Sales Representative. In this **MyLab Math** course, you have access to the most cutting-edge, innovative study solutions proven to increase students' success.

#### **Additional Teacher Resources**

Most of the teacher supplements and resources available for this text are available electronically for download at the Instructor Resource Center (IRC). Please go to the Pearson Global Editions site, https://media.pearsoncmg.com/intl/ge/abp/resources/ index.html, and select Instructor Resources. Once you register on the resources site, you will be able to access downloadable resources for your textbook.

The following supplements are available to registered adopters:

#### **Online Solutions Manual (Download Only)**

Provides complete solutions to all exercises, including Explorations, Quick Reviews, Exercises, Review Exercises, and Modeling Projects.

#### **Online Resource Manual (Download Only)**

Provides Major Concepts Review, Group Activity Worksheets, Sample Chapter Tests, Standardized Test Preparation Questions, and Contest Problems.

#### **Online Tests and Quizzes (Download Only)**

Provides two parallel tests per chapter, two quizzes for every three to four sections, two parallel midterm tests covering Chapters P–5, and two parallel end-of-year tests covering Chapters 6–11.

#### TestGen<sup>®</sup> (Download Only)

TestGen enables teachers to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing teachers to create multiple but equivalent versions of the same question or test with the click of a button. Teachers can also modify test bank questions or add new questions. Tests can be printed or administered online.

#### PowerPoint Slides (Download Only)

Features presentations written and designed specifically for this text, including figures, alternative examples, definitions, and key concepts.