Simulation

SEVENTH
EDITION

WITH ARENA

W. David Kelton | Nancy B. Zupick
Nathan J. Ivey

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Hill

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Seventh Edition

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His research interests and publications are in the probabilistic and statistical aspects of simulation, applications of simulation, and stochastic models. His papers have appeared in *Operations Research*, *Management Science*, the *INFORMS Journal on Computing*, *IIE Transactions*, *Naval Research Logistics*, *Military Operations Research*, the *European Journal of Operational Research*, *Simulation*, *Socio-Economic Planning Sciences*, the *Journal of Statistical Computation and Simulation*, and the *Journal of the American Statistical Association*, among others. In addition to the United States and Canada, he has spoken on simulation in Austria, Germany, Switzerland, The Netherlands, Belgium, Spain, Poland, Turkey, Chile, and South Korea.

He was editor-in-chief for the *INFORMS Journal on Computing* from 2000 to mid-2007, during which time the journal rose from unranked on the ISI Impact Factor to first out of 56 journals in the operations-research/management-science category. In addition, he has served as Simulation Area Editor for *Operations Research*, the *INFORMS Journal on Computing*, and *IIE Transactions*; Associate Editor of *Operations Research*, the *Journal of Manufacturing Systems*, and *Simulation*; and was Guest Co-Editor for a special simulation issue of *IIE Transactions*. Awards include the TIMS College on Simulation award for best simulation paper in *Management Science*, the IIE Operations Research Division Award, a Meritorious Service Award from *Operations Research*, the INFORMS College on Simulation Distinguished Service Award, and the INFORMS College on Simulation Outstanding Simulation Publication Award. He was President of the TIMS College on Simulation, and was the INFORMS co-representative to the Winter Simulation Conference Board of Directors from 1991 through 1999, serving as Board Chair for 1998. In 1987 he was Program Chair for the WSC, and in 1991 was General Chair; he is a founding Trustee of the WSC Foundation. He has worked on grants and consulting contracts from a number of corporations, foundations, and agencies. He has twice made it down a black-diamond run in the back bowls, both times upright on his skis.

Nancy B. Zupick is Manager of Arena Simulation at Rockwell Automation. She works with her staff to meet the simulation needs of Rockwell's clients across various industries. In addition to this role, she also works in product management and development, and helps manage the IISE/Rockwell Automation Student Simulation Contest and participates in marketing and sales activities.

Nancy received her bachelor's degree in industrial engineering from the University of Pittsburgh. It was at Pitt where she became hooked on simulation in Dr. Byron Gottfried's class and where she first became familiar with the SIMAN simulation language. She has worked for Rockwell Automation over the past twenty-five years in the Arena Simulation group and has never lost her passion for simulation and its application in industry.

When not immersed in the world of simulation, Nancy spends her time hanging out with her family. She enjoys gardening, cooking, and traveling, and occasionally dabbles in politics to reduce stress.

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Nathan earned both a Bachelor's and Master's degree in Industrial Engineering from the Pennsylvania State University. While at Penn State, Nathan first learned both SIMAN and Arena from Dr. Russell Barton, Dr. Dave Christy, Dr. Catherine Harmonosky, and Dr. Ben Kleindorfer. He thought he knew Arena when he took his first role at Systems Modeling. However, it was there that he really learned the "in's and out's" of Arena and simulation consulting.

When not simulating, Nathan enjoys spending time with his family, biking, or playing guitar in one of his many musical projects.

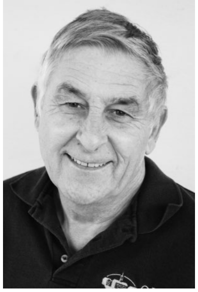
To those in the truly important arena of our lives:

Albert, Anna, Anne, Christie, and Molly, and Ridge

Aidan, Charity, Emma, Jenny, Michael, Mya, Noah, Sammy, Sean, Shelley, and Tierney

Adelaide and Lucinda

Ian and Anna



Dr. Randall P. Sadowski, who was our co-author on the first six editions of this book, passed away on May 17, 2014, shortly after work was finished on the sixth edition, in Columbus, Indiana.
A native of Toledo, Ohio, Randy earned BS and MS degrees in Mechanical Engineering from Ohio University and a PhD in Industrial Engineering from Purdue University. He served on the IE faculties at the University of Massachusetts and at Purdue before moving to the private sector, where he held a number of senior leadership positions with Systems Modeling Corporation and Rockwell Automation. A Fellow and Senior Member of the Institute of Industrial Engineers (IIE), Randy founded and, for many years, led the annual IIE/RS Student Simulation Contest. He also served IIE as VP-Systems Integration, as president at the chapter and division levels, and was General Chair of the 1990 Winter Simulation Conference. Dr. Sadowski co-authored two simulation textbooks, published in eight editions combined and translated into four other languages, and authored more than 65 articles and papers.
He was on the external advisory boards for IE departments at Ohio University, Lehigh University, and the University of Pittsburgh; he was a Distinguished Graduate of Ohio University.

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Contents

Preface

This seventh edition of *Simulation with Arena* has the same goal as the first six editions: to provide an introduction to simulation using Arena. It is intended as an entry-level simulation text, most likely in a first course on simulation at the undergraduate or beginning graduate level. However, material from the later chapters could be incorporated into a second graduate-level course. The book can also be used to learn simulation independent of a formal course (more specifically, by Arena users). The objective is to present the concepts and methods of simulation using Arena as a vehicle to help the reader reach the point of being able to carry out effective simulation modeling, analysis, and projects using the Arena simulation system. While we'll cover most of the capabilities of Arena, the book is not meant to be an exhaustive reference on the software, which is fully documented in its extensive online reference and help system.

Included in Appendix D are instructions on how to download the latest academic version of Arena and all the examples in the text. The website for this download and for the book in general is accessible through the Instructor Resources in Connect. Everything (including the Arena academic software and example files discussed in the book) is available from this site. We encourage all readers to visit this site to learn of any updates or errata for the book or example files, possible additional exercises, and other items of interest. At the time of this book's writing, the current version of Arena was 16.2 so the book is based on that. However, the book will continue to be useful for learning about later versions of Arena, the academic versions of which may be posted on the book's website as well for downloading. The site also contains material to support instructors who have adopted the book for use in class, including downloadable lecture slides and solutions to exercises; instructors who have adopted the book should contact their local McGraw Hill representative for authorization (see www.mhhe.com to locate local representatives). Software support is supplied only to the registered instructor via the instructions provided at the book's website: Software support is supplied to the registered instructor via the instructions provided on the book's website which can be accessed through Connect. Instructors adopting this book for classroom use will receive a free lab license from Rockwell Automation; please visit the Arena website, www.arenasimulation.com, for more information on this program or contact Arena Support at arena-support@ra.rockwell.com.

We've adopted an informal, tutorial writing style centered around carefully crafted examples to aid the beginner in understanding the ideas and topics presented. Ideally, readers would build simulation models as they read through the chapters. We start by having the reader develop simple, well-animated, high-level models, and then progress to advanced modeling and analysis. Statistical analysis is not treated as a separate topic, but is integrated into many of the modeling chapters, reflecting the joint nature of these activities in good simulation studies. We've also devoted the last two chapters to statistical issues and project planning to cover more advanced issues not treated in our modeling chapters. We believe that this approach greatly enhances the learning process by placing it in a more realistic and (frankly) less boring setting.

We assume neither prior knowledge of simulation nor computer-programming experience. We do assume basic familiarity with computing in general (files, folders, basic editing operations, etc.), but nothing advanced. A fundamental understanding of probability and statistics is needed, though we provide a self-contained refresher of these subjects in Appendices B and C. We also assume knowledge of basic derivative and integral calculus.

Here's a quick overview of the topics and organization. We start in Chapter 1 with a general introduction, a brief history of simulation, and modeling concepts. Chapter 2 addresses the simulation process using a simple simulation executed by hand and briefly discusses using spreadsheets to simulate very simple models (primarily static rather than dynamic simulations). In Chapter 3, we acquaint readers with Arena by examining a completed simulation model of the problem simulated by hand in Chapter 2, rebuilding it from scratch, going over the Arena user interface, and providing an overview of Arena's capabilities; we also provide a small case study illustrating how knowledge of just these basic building blocks of Arena allows one to address interesting and realistic issues.

Chapters 4 and 5 advance the reader's modeling skills by considering one "core" example per chapter, in increasingly complex versions to illustrate a variety of modeling and animation features; the statistical issue of selecting input probability distributions is also covered in Chapter 4 using the Arena Input Analyzer, and a non-queueing (inventory) model is at the end of Chapter 5.

Chapter 6 uses one of the models in Chapter 5 to illustrate the basic Arena capabilities of statistical analysis of output, including single-system analysis, comparing multiple scenarios (configurations of a model), and searching for an optimal scenario; this material uses the Arena Output and Process Analyzers, as well as OptQuest for Arena.

In Chapter 7, we introduce another "core" model, again in increasingly complex versions, and then use it to illustrate statistical analysis of long-run (steady-state) simulations. Alternate ways in which simulated entities can move around is the subject of Chapter 8, including material-handling capabilities, building on the models in Chapter 7. Chapter 9 digs deeper into Arena's extensive modeling constructs, using a sequence of small, focused models to present a wide variety of special-purpose capabilities; this is for more advanced simulation users and would probably not be covered in a beginning course.

In Chapter 10, we describe a number of topics in the area of customizing Arena and integrating it with other applications like spreadsheets and databases; this includes using Visual Basic for Applications (VBA) with Arena. Also included in this chapter is an introduction to Arena's string functionality as well as a brief overview of Arena's new Visual Designer Application. Chapter 11 shows how Arena can handle continuous and combined discrete/continuous models, such as fluid flow. Chapter 12 covers more advanced statistical concepts underlying and applied to simulation analysis, including random-number generators, variate and process generation, variance-reduction techniques, sequential sampling, and designing simulation experiments. Chapter 13 provides a broad overview of the simulation process and discusses more specifically the issues of managing and disseminating a simulation project.

Appendix A describes a complete modeling specification from a project for *The Washington Post* newspaper. Appendix B gives a complete but concise review of the basics of probability and statistics couched in the framework of their role in simulation modeling and analysis. The probability distributions supported by Arena are detailed in Appendix C. Installation instructions for the Arena academic software can be found in Appendix D. All references are collected in a single References section at the end of the book. The index is extensive, to aid readers in locating topics and seeing how they relate to each other; the index includes authors cited.

As mentioned, the presentation is in "tutorial style," built around a sequence of carefully crafted examples illustrating concepts and applications, rather than in the conventional style of stating concepts first and then citing examples as an afterthought. So it probably makes sense to read (or teach) the material essentially in the order presented. A one-semester or one-quarter first course in simulation could cover all the material in Chapters 1–8, including the statistical material. Time permitting, selected modeling and computing topics from Chapters 9–11 could be included, or some of the more advanced statistical issues from Chapter 12, or the project-management material from Chapter 13, according to the instructor's tastes. A second course in simulation could assume most of the material in Chapters 1–8, then cover the more advanced modeling ideas in Chapters 9–11, followed by topics from Chapters 12 and 13. For self-study, we'd suggest going through Chapters 1–6 to understand the basics, getting at least familiar with Chapters 7 and 8, then regarding the rest of the book as a source for more advanced topics and reference. Regardless of what's covered, and whether the book is used in a course or independently, it will be helpful to follow along in Arena on a computer while reading this book.

The academic version of Arena (see Appendix D for instructions on downloading and installing the software), has all the modeling and analysis capabilities of the complete commercial version, but limits model size. All the examples in the book, as well as all the exercises at the ends of the chapters, will run with this academic version of Arena. The download also contains files for all the example models in the book, as well as other support materials. This software can be installed on any university computer as well as on students' computers. It is intended for use in conjunction with this book for the purpose of learning simulation and Arena. It is not authorized for use in commercial environments.

In revising the book to this seventh edition, several important aspects changed. We've moved to Arena version 16.2 (from version 14.5 in the prior edition), which contains many new and useful features; all text and screenshots have been accordingly updated, as have all of the example files (in the Book Examples folder that's available for download as a *.zip*-file archive online through Connect). There are now additional end-of-chapter Exercises, but we've retained all of the prior Exercises using the same numbering as before so the new Exercises just continue in the numbering scheme within each chapter; many prior Exercises have been updated and improved. As before, solutions to the Exercises are available to instructors who've adopted the book for use in a formal course, as are PowerPoint slides that have also been updated to go along with this seventh edition. The most extensive changes in the book are in Chapters 3 and 10, which discuss the new Arena 16.2 interface and capabilities. Appendix D, on downloading and installing the academic version of Arena 16.2 has also been mostly rewritten to describe the new and simplified procedures; Rockwell Automation will again provide the academic version free of charge. Of course, all known errata from the prior edition have been corrected and implemented.

As with any labor like this, there are a lot of people and institutions that supported us in a lot of different ways throughout the years. This support has resulted in the many editions of this textbook. While those people are too many to name, know that we appreciated your time and consideration.

Rockwell Automation provided resources in the form of time, software, hardware, technical assistance, and moral encouragement; we would particularly like to thank the Arena development team—Mark Glavach, Cynthia Kasales, Tomasz Glowacki, Krzysztof Piwowarczyk, Michal Ignaczak, Ivo Peterka, Zdenek Kodejs, Martin Skalnik, Martin Paulicek, Hynek Frauenberg, and Judy Jordan—as well as Melanie Barker, Tyler DiFrischia, Dawn Cannister, Gail Kenny, and Darrell Starks.

A special thanks goes out to Susan Strickling in the Arena Support Group for assisting with the solutions to the end-of-chapters Exercises and migrating work into the new version of Arena.

We are also grateful to Gary Lucke and Olivier Girod of *The Washington Post* for allowing us to include a simulation specification that was developed for them by Rockwell Automation as part of a larger project. Special thanks go to Peter Kauffman for his designs of the covers of the earlier editions, and to Jim McClure for his cartoon and illustration design. And we appreciate the skillful motivation and gentle nudging by our editors at McGraw Hill, Amity Watts and Erin Kamm. Reviewers of earlier editions, including Bill Harper, Mansooreh Mollaghasemi, Barry Nelson, Ed Watson, and King Preston White Jr., provided extremely valuable input and help, ranging from overall organization and content all the way to the downright subatomic. Thanks are also due to the many individuals who have used part or all of the early material in classes (as well as to their students who were subjected to early drafts), plus a host of other folks who provided all kinds of input, feedback and help: Christos Alexopoulos, Ken Bauer, Diane Bischak, Sherri Blaszkiewicz, Eberhard Blümel, Mike Branson, Jeff Camm, Colin Campbell, John Charnes, Chun-Hung Chen, Hong Chen, Jack Chen, Russell Cheng, Christopher Chung, Frank Ciarallo, John J. Clifford, Mary Court, Tom Crowe, Halim Damerdji, Pat Delaney, Mike Dellinger, Darrell Donahue, Ken Ebeling, Neil Eisner, Gerald Evans, Steve Fisk, Michael Fu, Shannon Funk, Fred Glover, Dave Goldsman, Byron Gottfried, Frank Grange, Don Gross, John Gum, Nancy Markovitch Gurgiolo,Tom Gurgiolo, Jorge Haddock, Bill Harper, Joe Heim, Michael Howard, Arthur Hsu, Alberto Isla, Eric Johnson, Elena Joshi, Keebom Kang, Parastu Kasaie, Elena Katok, Jim Kelly, Teri King, Gary Kochenberger, Patrick Koelling, David Kohler, Wendy Krah, Bradley Kramer, Michael Kwinn Jr., Averill Law, Larry Leemis, Marty Levy, Vladimir Leytus, Bob Lipset, Tom Lucas, Gerald Mackulak, Deb Mederios, Brian Melloy, Mansooreh Mollaghasemi, Ed Mooney, Jack Morris, Jim Morris, Charles Mosier, Marvin Nakayama, Dick Nance, Barry Nelson, James Patell, Cecil Peterson, Dave Pratt, Mike

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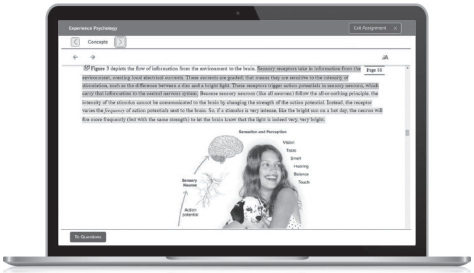

Figure 3 depicts the flow of information from the environment to the brain. Sensory receptors take in information from the environment, creating local electrical currents. These currents are graded; that means they are sensitive to the intensity of stimulation, such as the difference between a dim and a bright light. These receptors trigger action potentials in sensory neurons, which carry that information to the central nervous system. Because sensory neurons (like all neurons) follow the all-or-nothing principle, the intensity of the stimulus cannot be communicated to the brain by changing the strength of the action potential. Instead, the receptor varies the *frequency* of action potentials sent to the brain. So, if a stimulus is very intense, like the bright sun on a hot day, the neuron will fire more frequently (but with the same strength) to let the brain know that the light is indeed very, very bright.

Laptop: Getty Images; Woman/dog: George Doyle/Getty Images

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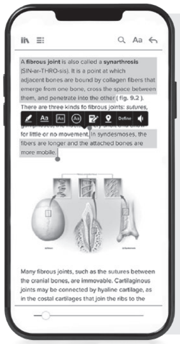

A **fibrous joint** is also called a **synarthrosis** (SIN-ar-THRO-sis). It is a point at which adjacent bones are bound by collagen fibers that emerge from one bone, cross the space between them, and penetrate into the other (fig. 9.2). There are three kinds fo fibrous joints: *sutures,*

for little or no movement. In syndesmoses, the fibers are longer and the attached bones are more mobile.

Many fibrous joints, such as the sutures between the cranial bones, are immovable. Cartilaginous joints may be connected by hyaline cartilage, as in the costal cartilages that join the ribs to the

iPhone: Getty Images

> *"I really liked this app—it made it easy to study when you don't have your text-book in front of you."*
>
> - Jordan Cunningham,
> Eastern Washington University

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CHAPTER 1

What Is Simulation?

Simulation refers to a broad collection of methods and applications to mimic the behavior of real systems, usually on a computer with appropriate software. In fact, "simulation" can be an extremely general term since the idea applies across many fields, industries, and applications. These days, simulation is more popular and powerful than ever since computers and software are better than ever.

This book gives you a comprehensive treatment of simulation in general and the Arena simulation software in particular. We cover the general idea of simulation and its logic in Chapters 1 and 2 (including a bit about using spreadsheets to simulate) and Arena in Chapters 3–9. We don't, however, intend for this book to be a complete reference on everything in Arena (that's what the help systems and Arena Product Manuals and User's Guides included with the software are for), or on everything on the statistical design and analysis of simulations (there are whole books for that, though we do cover some of this throughout, especially in Chapter 6, Section 7.2, Chapter 12, and in Appendices B and C). In Chapter 10, we show you how to integrate Arena with external files and other applications and give an overview of some advanced Arena capabilities. In Chapter 11, we introduce you to continuous and combined discrete/continuous modeling with Arena. Chapters 12 and 13 cover issues related to planning and interpreting the results of simulation experiments, as well as managing a simulation project. Appendix A is a detailed account of a simulation project carried out for *The Washington Post* newspaper. Appendix B provides a quick review of probability and statistics necessary for simulation. Appendix C describes Arena's probability distributions, and Appendix D provides software installation instructions. After reading this book, you should be able to model systems with Arena and carry out effective and successful simulation studies.

This chapter touches on the general notion of simulation. In Section 1.1, we describe general ideas about how you might study models of systems and give examples of where simulation has been useful. Section 1.2 contains more specific information about simulation and its popularity, mentions some good things (and one bad thing) about simulation, and attempts to classify the many different kinds of simulations that people do. In Section 1.3, we talk a little bit about software options. Finally, Section 1.4 traces changes over time in how and when simulation is used. After reading this chapter, you should have an appreciation for where simulation fits in, the kinds of things it can do, and how Arena might be able to help you do them.

1.1 Modeling

Simulation, like most analysis methods, involves systems and models of them. So in this section, we give you examples of systems and describe options for studying them to learn about their behavior, perhaps improve them, or design them if they don't yet exist.

1.1.1 What's Being Modeled?

Computer simulation deals with models of systems. A *system* is a facility or process, either actual or planned, such as:

- A manufacturing plant with machines, people, transport devices, conveyor belts, and storage space.
- A bank with different kinds of customers, servers, and facilities like teller windows, automated teller machines (ATMs), loan desks, and safety deposit boxes.
- An airport with departing passengers checking in, going through security, going to the departure gate, and boarding; departing flights contending for push-back tugs and runway slots; arriving flights contending for runways, gates, and arrival crew; arriving passengers moving to baggage claim and waiting for their bags; and the baggage-handling system dealing with delays, security issues, and equipment failure.
- A distribution network of plants, warehouses, and transportation links.
- An emergency facility in a hospital, including personnel, rooms, equipment, supplies, and patient transport.
- A field-service operation for appliances or office equipment, with potential customers scattered across a geographic area, service technicians with different qualifications, trucks with different parts and tools, and a central depot and dispatch center.
- A computer network with servers, clients, storage systems, and networking capabilities.
- A freeway system of road segments, interchanges, controls, and traffic.
- A central insurance claims office where a lot of documents are received, reviewed, filed, and transmitted by people and computers.
- A criminal-justice system of courts, judges, support staff, probation officers, parole agents, defendants, plaintiffs, convicted offenders, and schedules.
- A chemical-products plant with storage tanks, pipelines, reactor vessels, and railway tanker cars in which to ship the finished product.
- A fast-food restaurant with different types of staff, customers, and equipment.
- A supermarket with inventory control, checkout, and customer service.
- A theme park with rides, stores, restaurants, workers, guests, and parking lots.
- The response of emergency personnel to a catastrophic event.
- A network of shipping ports including ships, containers, cranes, and landside transport.
- A military operation including supplies, logistics, and combat engagement.

People often study a system to measure its performance, improve its operation, or improve its design if it doesn't exist. Managers or controllers of a system might also like to have a readily available aid for day-to-day operations, like help in deciding what to do in a factory if an important machine goes down.

We're even aware of managers who requested that simulations be constructed but didn't really care about the final results. Their primary goal was to focus attention on understanding how their system worked. Often simulation analysts find that the process of defining how the system works, which must be done before you can start developing the simulation model, provides great insight into what changes need to be made. Part of this is due to the fact that rarely is there one individual responsible for understanding how an entire system works. There are experts in machine design, material handling, processes, and so on, but not in the day-to-day operation of the system. So as you read on, be aware that simulation is much more than just building a model and conducting a statistical experiment. There is much to be learned at each step of a simulation project, and the decisions you make along the way can greatly affect the significance of your findings.

1.1.2 How About Just Playing with the System?

It might be possible to experiment with the actual physical system. For instance:

- Some cities have installed entrance-ramp traffic lights on their freeway systems to experiment with different sequencing to find settings that make rush hour as smooth and safe as possible.
- A supermarket manager might try different policies for inventory control and checkout-personnel assignment to see what combinations seem to be most profitable and provide the best service.
- An airline could test the expanded use of automated check-in kiosks (with employees urging passengers to use them) to see if this speeds check-in.
- A computer facility can experiment with different network layouts and task-execution priorities to see how they affect processor utilization and throughput.

This approach certainly has its advantages. If you can experiment directly with the system and know that nothing else about it will change significantly, then you're unquestionably looking at the right thing and needn't worry about whether a model or proxy for the system faithfully mimics it for your purposes.

1.1.3 Sometimes You Can't (or Shouldn't) Play with the System

In many cases, it's just too difficult, costly, or downright impossible to do physical studies on the system itself.

- Obviously, you can't experiment with alternative layouts of a factory if it's not yet built.
- Even in an existing factory, it might be prohibitively costly to change to an experimental layout that might not work out in the end.
- It would be hard to run twice as many customers through a bank to see the effect of closing a nearby branch.
- Trying a new check-in procedure at an airport might initially cause a lot of people to miss their flights if there are unforeseen problems with the new procedure.
- Fiddling around with emergency room staffing in a hospital clearly won't do.

In these situations, you might build a *model* to serve as a stand-in for studying the system and ask pertinent questions about what *would* happen in the system *if* you did this or that, or *if* some situation beyond your control were to develop. *Nobody gets hurt, and your freedom to try wide-ranging ideas with the model could uncover attractive alternatives that you might not have been able to try with the real system.*

However, you have to build models carefully and with enough detail so that what you learn about the model will never[1] be different from what you would have learned about the system by playing with it directly. This is called model *validity*, and we'll have more to say about it later, in Chapter 13.

1.1.4 Physical Models

There are lots of different kinds of models. Maybe the first thing the word evokes is a physical replica or scale model of the system, sometimes called an *iconic* model. For instance:

- People have built *tabletop* models of material handling systems that are miniature versions of the facility, not unlike electric train sets, to consider the effect on performance of alternative layouts, vehicle routes, and transport equipment.
- A full-scale version of a fast-food restaurant placed inside a warehouse to experiment with different service procedures was described by Swart and Donno (1981).
- Simulated control rooms have been developed to train operators for nuclear power plants.
- Physical flight simulators are widely used to train pilots. There are also flight-simulation computer programs, with which you may be familiar in game form, that represent purely logical models executing inside a computer. Further, physical flight simulators might have computer screens to simulate airport approaches, so they have elements of both physical and computer-simulation models.

Although iconic models have proven useful in many areas, we won't consider them.

1.1.5 Logical (or Mathematical) Models

Instead, we'll consider *logical* (or *mathematical*) models of systems. Such a model is just a set of approximations and assumptions, both structural and quantitative, about the way the system does or will work.

A logical model is usually represented in a computer program that's exercised to address questions about the model's behavior; if your model is a valid representation of your system, you hope to learn about the system's behavior too. And since you're dealing with a mere computer program rather than the actual system, it's usually easy, cheap, and fast to get answers to a lot of questions about the model and system by simply manipulating the program's inputs and form. Thus, you can make your mistakes on the computer where they don't count, rather than for real where they do. As in many other fields, recent dramatic increases in computing power (and decreases in computing costs) have impressively advanced your ability to carry out computer analyses of logical models.

1.1.6 What Do You Do with a Logical Model?

After making the approximations and stating the assumptions for a valid logical model of the target system, you need to find a way to deal with the model and analyze its behavior.

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[1] Well, hardly ever.

If the model is simple enough, you might be able to use traditional mathematical tools like queueing theory, differential-equation methods, or something like linear programming to get the answers you need. This is a nice situation since you might get fairly simple formulas to answer your questions, which can easily be evaluated numerically; working with the formula (for instance, taking partial derivatives of it with respect to controllable input parameters) might provide insight itself. Even if you don't get a simple closed-form formula, but rather an algorithm to generate numerical answers, you'll still have exact answers (up to round-off, anyway) rather than estimates that are subject to uncertainty.

However, most systems that people model and study are pretty complicated, so that *valid* models[2] of them are pretty complicated too. For such models, there may not be exact mathematical solutions worked out, which is where simulation comes in.

1.2 Computer Simulation

Computer simulation refers to methods for studying a wide variety of models of real-world systems by numerical evaluation using software designed to imitate the system's operations or characteristics, often over time. From a practical viewpoint, simulation is the process of designing and creating a computerized model of a real or proposed system for the purpose of conducting numerical experiments to give us a better understanding of the behavior of that system for a given set of conditions. Although it can be used to study simple systems, the real power of this technique is fully realized when we use it to study complex systems.

While simulation may not be the only tool you could use to study the model, it's frequently the method of choice. The reason for this is that the simulation model can be allowed to become quite complex, if needed to represent the system faithfully, and you can still do a simulation analysis. Other methods may require stronger simplifying assumptions about the system to enable an analysis, which might bring the validity of the model into question (see Lucas *et al.,* 2015).

1.2.1 Popularity and Advantages

Over the last several decades, simulation has been consistently reported as the most popular operations-research tool:

- Rasmussen and George (1978) asked M.S. graduates from the Operations Research Department at Case Western Reserve University (of which there are many since that department was founded a long time ago) about the value of methods after graduation. The first four methods were *statistical analysis*, *forecasting*, *systems analysis*, and *information systems*, all of which are very broad and general categories. Simulation was next, and ranked higher than other more traditional operations-research tools like linear programming and queueing theory.

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[2] You can always build a simple (maybe simplistic) model of a complicated system, but there's a good chance that it won't be valid. If you go ahead and analyze such a model, you may be getting nice, clean, simple answers to the wrong questions. This is sometimes called a Type III Error—working on the wrong problem (statisticians have already claimed Type I and Type II Errors). Lucas *et al.* (2015) study this in some detail, and provide an example of just how wrong models can be if they're over-simplified just to get an analytical, closed-form mathematical solution.

- Thomas and DaCosta (1979) gave analysts in 137 large firms a list of tools and asked them to check off which ones they used. Statistical analysis came in first, with 93% of the firms reporting that they use it (it's hard to imagine a large firm that wouldn't), followed by simulation (84%). Again, simulation came in higher than tools like linear programming, PERT/CPM, inventory theory, and nonlinear programming.
- Shannon, Long, and Buckles (1980) surveyed members of the Operations Research Division of the American Institute of Industrial Engineers (now the Institute of Industrial and Systems Engineers) and found that among the tools listed, simulation ranked first in utility and interest. Simulation was second in familiarity, behind linear programming, which might suggest that simulation should be given stronger emphasis in academic curricula.
- Forgionne (1983); Harpell, Lane, and Mansour (1989); and Lane, Mansour, and Harpell (1993) all report that, in terms of utilization of methods by practitioners in large corporations, statistical analysis was first and simulation was second. Again, though, academic curricula seem to be behind since linear programming was more frequently *taught*, as opposed to being *used* by practitioners, than was simulation.
- Morgan (1989) reviewed many surveys of the preceding type and reported that "heavy" use of simulation was consistently found. Even in an industry with the lowest reported use of operations-research tools (motor carriers), simulation ranked first in usage.
- Powers (2012) carried out a thorough survey of the research literature in operations research and found literally exponential growth over the past several decades in the number of papers applying simulation or developing simulation methods, whereas the growth in the number of papers on more traditional operations-research tools has been far slower.

The main reason for simulation's popularity is its ability to deal with complicated models of correspondingly complicated systems. This makes it a versatile and powerful tool. Another reason for simulation's increasing popularity is the obvious improvement in performance/price ratios of computer hardware and multi-core capabilities, making the cost of building and running simulation models inexpensive and fast. Finally, advances in simulation software power, flexibility, and ease of use have moved the approach from the realm of tedious and error-prone, low-level programming to the arena of quick and valid decision making.

Our guess is that simulation's popularity and effectiveness are now even greater than reported in the older of the surveys described here, precisely due to these advances in computer hardware and software.

1.2.2 The Bad News

However, simulation isn't *quite* paradise.

Because many real systems are affected by uncontrollable and random inputs, many simulation models involve random, or *stochastic*, input components, causing their output to be random too. For example, a model of a distribution center would have arrivals, departures, and lot sizes arising randomly according to particular probability distributions, which will propagate through the model's logic to cause output performance measures like throughput and cycle times to be random as well. So running a stochastic simulation once is like performing a random physical experiment once, or watching the distribution center for one day—you'll probably see something different next time, even if you don't change anything yourself. In many simulations, as the time frame becomes longer (like months instead of a day), most results averaged over the run will tend to settle down and become less variable, but it can be hard to determine how long is "long enough" for this to happen. Moreover, the model or study might dictate that the simulation stop at a particular point (for instance, a bank is open from 9 to 5), so running it longer just to calm the output is inappropriate.

Thus, you have to think carefully about designing and analyzing simulation experiments to take account of this uncertainty in the results, especially if the appropriate time frame for your model is relatively short. We'll return to this idea repeatedly in the book and illustrate proper statistical design and analysis tools, some of which are built into Arena, but others you have to worry about yourself.

Even though simulation output may be uncertain, we can deal with, quantify, and reduce this uncertainty. You might be able to get rid of the uncertainty completely by making a lot of over-simplifying assumptions about the system; this would get you a nice, simple model that will produce nice, non-random results. Unfortunately, though, such an over-simplified model will probably not be a *valid* representation of the system, and the error due to such model invalidity is impossible to measure or reliably reduce (again, see Lucas *et al.,* 2015). For our money, we'd prefer an approximate answer to the right problem rather than an exact answer to the wrong problem (remember the Type III Error?), especially when we can readily sharpen the simulation approximation with more computing or better design of the simulation experiments (see Chapter 6, as well as Sections 7.2, 12.4, and 12.6).

1.2.3 Different Kinds of Simulations

There are a lot of ways to classify simulation models, but one useful way is along these three dimensions:

1. **Static vs. Dynamic:** Time doesn't play a natural role in static models but does in dynamic models. The Buffon needle problem, described in Section 1.3.1, is a static simulation. The small manufacturing model described in Chapters 2 and 3 is a dynamic model. Most operational models are dynamic; Arena was designed with them in mind, so our primary focus will be on such models (except for Section 2.7.1, which develops a static model using only a spreadsheet).

2. **Continuous vs. Discrete:** In a continuous model, the state of the system can change continuously over time; an example would be the level of a reservoir as water flows in and is let out, and as precipitation and evaporation occur. In a discrete model, though, change can occur only at separated points in time, such as a manufacturing system with parts arriving and leaving at specific times, machines' going down and coming back up at specific times, and breaks for workers. You can have elements of both continuous and discrete change in the same model,

which are called *mixed continuous-discrete models*; an example might be a refinery with continuously changing pressure inside vessels and discretely occurring shutdowns. Arena can handle continuous, discrete, and mixed models; our focus will be on discrete models for most of the book, though Chapter 11 discusses continuous and mixed models.

3. **Deterministic vs. Stochastic:** Models that have no random input are deterministic; a strict appointment-book operation with fixed service times is an example. Stochastic models, on the other hand, operate with at least some inputs' being random—like a bank with randomly arriving customers requiring varying service times. A model can have both deterministic and random inputs in different components; which elements are modeled as deterministic and which as random are issues of modeling realism. Arena easily handles deterministic and stochastic inputs to models and provides many different probability distributions and processes that you can use to represent the random inputs. Since we feel that at least some element of uncertainty is usually present in reality, most of our illustrations involve random inputs somewhere in the model. As noted earlier, though, stochastic models produce uncertain output, which is a fact you must consider carefully in designing and interpreting the runs in your project.

1.3 How Simulations Get Done

If you've determined that a simulation of some sort is appropriate, you then have to decide how to carry it out. In this section, we discuss options for running a simulation, including software.

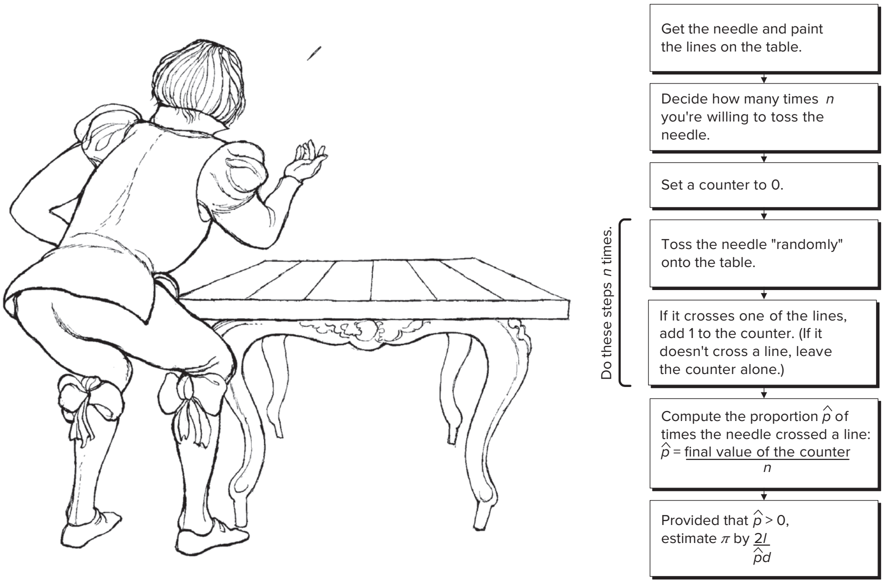

Figure 1-1. The Buffon Needle Problem

1.3.1 By Hand

In the beginning, people really *did* do simulations by hand (we'll show you just one, which is painful enough, in Section 2.4).

For instance, around 1733 a fellow by the name of Georges Louis Leclerc (who later was invited into the nobility, due no doubt to his simulation prowess, as Comte de Buffon) described an experiment to estimate the value of π. If you toss a needle of length l onto a table painted with parallel lines spaced d apart (d must be $\geq l$), it turns out that the needle will cross a line with probability $p = 2l/(\pi d)$. So Figure 1-1 shows a simulation experiment to estimate the value of π. (Don't try this at home, or at least not with a big needle.)

Though this experiment may seem pretty simple (probably even silly) to you, there are some aspects of it that are common to most simulations:

- The purpose is to estimate something (in this case, π) whose value would be hard to compute exactly (okay, maybe in 1733 that was true).
- The estimate we get at the end is not going to be exactly right; that is, it has some error associated with it, and it might be nice to get an idea of how large that error is likely to be.
- It seems intuitive that the more tosses we make (that is, the bigger n is), the smaller the error is likely to be and thus the better the estimate is likely to be.
- In fact, you could do a *sequential* experiment and just keep tossing until the probable error is small enough for you to live with, instead of deciding on the number n of tosses beforehand.
- You might be able to reduce the error without increasing the number of tosses if you invest a little up-front work. Weld a second needle of the same length to the first so they cross at right angles at their midpoints; such a weapon is called a *Buffon cross*. Leave the lines on the table alone. On each toss, record separately whether each needle crosses a line (it could be that they both cross, neither crosses, or just one but not the other crosses), and get two different estimates of π. It's intuitive (and, happily, true) that whether one needle crosses a line is negatively correlated with whether the other one does, so the two estimates of π will be negatively correlated with each other. The average of the two estimates is also an unbiased estimate of π, but will have less variance than a single-needle estimate from the same number of tosses since it's likely that if one estimate is high, the other one will be low. (This is a physical analog of what's called a *variance-reduction technique*, specifically *antithetic variates*, and is discussed in Section 12.4.2. It seems like some kind of cheat or swindle, but it's really a fair game.)

We'll come back to these kinds of issues as we talk about more interesting and helpful simulations. (For more on the Buffon needle problem, as well as other such interesting historical curiosities, see Morgan, 1984.)

In the 1920s and 1930s, statisticians began using random-number machines and tables in numerical experiments to help them develop and understand statistical theory. For instance, Walter A. Shewhart (the quality-control pioneer) did numerical experiments by drawing numbered chips from a bowl to study the first control charts.