AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

ICE-EM MATHEMATICS 10 &10A

THIRD EDITION

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INTERNATIONAL CENTRE OF EXCELLENCE FOR EDUCATION IN MATHEMATICS

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Preface

ICE-EM Mathematics Third Edition is a series of textbooks for students in years 5 to 10 throughout Australia who study the Australian Curriculum and its state variations.

The program and textbooks were developed in recognition of the importance of mathematics in modern society and the need to enhance the mathematical capabilities of Australian students. Students who use the series will have a strong foundation for work or further study.

Background

The International Centre of Excellence for Education in Mathematics (ICE-EM) was established in 2004 with the assistance of the Australian Government and is managed by the Australian Mathematical Sciences Institute (AMSI). The Centre originally published the series as part of a program to improve mathematics teaching and learning in Australia. In 2012, AMSI and Cambridge University Press collaborated to publish the Second Edition of the series to coincide with the introduction of the Australian Curriculum, and we now bring you the Third Edition.

The series

ICE-EM Mathematics Third Edition provides a progressive development from upper primary to middle secondary school. The writers of the series are some of Australia's most outstanding mathematics teachers and subject experts. The textbooks are clearly and carefully written, and contain background information, examples and worked problems.

For the Third Edition, the series has been carefully edited to present the content in a more streamlined way without compromising quality. There is now one book per year level and the flow of topics from chapter to chapter and from one year level to the next has been improved.

The year 10 textbook incorporates all material for the 10A course, and selected topics in earlier books carefully prepare students for this. *ICE-EM Mathematics Third Edition* provides excellent preparation for all of the Australian Curriculum's year 11 and 12 mathematics courses.

For the Third Edition, *ICE-EM Mathematics* now comes with an Interactive Textbook: a cutting-edge digital resource where all textbook material can be answered online (with students' working-out), additional quizzes and features are included at no extra cost. See 'The Interactive Textbook and Online Teaching Suite' on page xiii for more information.

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Author biographies

Peter Brown

Peter Brown studied Pure Mathematics and Ancient Greek at Newcastle University, and completed postgraduate degrees in each subject at the University of Sydney. He worked for nine years as a mathematics teacher in NSW State schools. Since 1990, he has taught Pure Mathematics at the School of Mathematics and Statistics at the University of New South Wales (UNSW). He was appointed Director of First Year Studies at UNSW from 2011 to 2015. He specialises in Number Theory and History of Mathematics and has published in both areas. Peter regularly speaks at teacher inservices, Talented Student days and Mathematics Olympiad Camps. In 2008 he received a UNSW Vice Chancellor's Teaching Award for educational leadership.

Michael Evans

Michael Evans has a PhD in Mathematics from Monash University and a Diploma of Education from La Trobe University. He currently holds the honorary position of Senior Consultant at the Australian Mathematical Sciences Institute at the University of Melbourne. He was Head of Mathematics at Scotch College, Melbourne, and has also taught in public schools and in recent years has returned to classroom teaching. He has been very involved with curriculum development at both state and national levels. In 1999, Michael was awarded an honorary Doctor of Laws by Monash University for his contribution to mathematics education, and in 2001 he received the Bernhard Neumann Award for contributions to mathematics enrichment in Australia.

Garth Gaudry

Garth Gaudry was Head of Mathematics at Flinders University before moving to UNSW, where he became Head of School. He was the inaugural Director of AMSI before he became the Director of AMSI's International Centre of Excellence for Education in Mathematics. Previous positions include membership of the South Australian Mathematics Subject Committee and the Eltis Committee appointed by the NSW Government to enquire into Outcomes and Profiles. He was a life member of the Australian Mathematical Society and Emeritus Professor of Mathematics, UNSW.

David Hunt

David Hunt graduated from the University of Sydney in 1967 with an Honours degree in Mathematics and Physics, then obtained a master's degree and a doctorate from the University of Warwick. He was appointed to a lectureship in Pure Mathematics at UNSW in early 1971, where he is currently an honorary Associate Professor. David has taught courses in Pure Mathematics from first year to master's level and was Director of First Year Studies in Mathematics for five years. Many of David's activities outside UNSW have centred on the Australian Mathematics Trust. These contributions as well as those to the International Mathematical Olympiad movement were recognised by the award of the Paul Erdos medal in 2016.

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Robert McLaren

Robert McLaren graduated from the University of Melbourne in 1978 with a Bachelor of Science (Hons) and a Diploma of Education. He commenced his teaching career in 1979 at The Geelong College and has taught at a number of Victorian Independent Schools throughout his career. He has been involved in textbook writing, curriculum development and VCE examination setting and marking during his teaching life. He has taught mathematics at all secondary levels and has a particular interest in problem solving. Robert is currently Vice Principal at Scotch College in Melbourne.

Bill Pender

Bill Pender has a PhD in Pure Mathematics from Sydney University and a BA (Hons) in Early English from Macquarie University. After a year at Bonn University, he taught at Sydney Grammar School from 1975 to 2008, where he was Subject Master for many years. He has been involved in the development of NSW Mathematics syllabuses since the early 1990s, and was a foundation member of the Education Advisory Committee of AMSI. He has also lectured and tutored at Sydney University and at UNSW, and given various inservice courses. Bill is the lead author of the NSW calculus series *Cambridge Mathematics.*

Brian Woolacott

Brian Woolacott graduated from the University of Melbourne in 1978 with a Bachelor of Science and a Diploma of Education. In 1979 he started his teaching career at Scotch College, Melbourne, and during his career he has taught at all secondary levels. For 13 years, Brian was the Co-ordinator of Mathematics for Years 9 and 10, and during this time he was involved in co-authoring a number of textbooks for the Year 9 and 10 levels. Brian is currently the Dean of Studies at Scotch College.

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How to use this resource

The textbook

Each chapter in the textbook addresses a specific Australian Curriculum content strand and set of sub-strands. The exercises within chapters take an integrated approach to the concept of proficiency strands, rather than separating them out. Students are encouraged to develop and apply Understanding, Fluency, Problem-solving and Reasoning skills in every exercise.

The series places a strong emphasis on understanding basic ideas, along with mastering essential technical skills. Mental arithmetic and other mental processes are major focuses, as is the development of spatial intuition, logical reasoning and understanding of the concepts.

Problem-solving lies at the heart of mathematics, so *ICE-EM Mathematics* gives students a variety of different types of problems to work on, which help them develop their reasoning skills. Challenge exercises at the end of each chapter contain problems and investigations of varying difficulty that should catch the imagination and interest of students. Further, two 'Review and Problem-solving' chapters in each 7–10 textbook contain additional problems that cover new concepts for students who wish to explore the subject even further.

The Interactive Textbook and Online Teaching Suite

Included with the purchase of the textbook is the Interactive Textbook. This is the online version of the textbook and is accessed using the 16-character code on the inside cover of this book.

The Online Teaching Suite is the teacher version of the Interactive Textbook and contains all the support material for the series, including tests, worksheets, skillsheets, curriculum documentation and more.

For more information on the Interactive Textbook and Online Teaching Suite, see page xiii.

The Interactive Textbook and Online Teaching Suite are delivered on the *Cambridge HOTmaths* platform, providing access to a world-class Learning Management System for testing, task management and reporting. They do not provide access to the *Cambridge HOTmaths* stand-alone resource that you or your school may have used previously. For more information on this resource, contact Cambridge University Press.

AMSI's TIMES and SAM modules

The TIMES and SAM web resources were developed by the *ICE-EM Mathematics* author team at AMSI and are written around the structure of the Australian Curriculum. These resources have been mapped against your *ICE-EM Mathematics* book and are available to teachers and students via the AMSI icon on the Dashboard of the Interactive Textbook and Online Teaching Suite.

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The Interactive Textbook and the Online Teaching Suite

Interactive Textbook

The Interactive Textbook is the online version of the print textbook and comes included with purchase of the print textbook. It is accessed by first activating the code on the inside cover. It is easy to navigate and is a valuable accompaniment to the print textbook.

Students can show their working

All textbook questions can be answered online within the Interactive Textbook. Students can show their working for each question using either the Draw tool for handwriting (if they are using a device with a touch-screen), the Type tool for using their keyboard in conjunction with the pop-up symbol palette, or by importing a file using the Import tool.

Once a student has completed an exercise they can save their work and submit it to the teacher, who can then view the student's working and give feedback to the student, as they see appropriate.

Auto-marked quizzes

The Interactive Textbook also contains material not included in the textbook, such as a short auto-marked quiz for each section. The quiz contains 10 questions which increase in difficulty from question 1 to 10 and cover all proficiency strands. There is also space for the student to do their working underneath each quiz question. The auto-marked quizzes are a great way for students to track their progress through the course.

Additional material for Year 5 and 6

For Years 5 and 6, the end-of-chapter Challenge activities as well as a set of Blackline Masters are now located in the Interactive Textbook. These can be found in the 'More resources' section, accessed via the Dashboard, and can then easily be downloaded and printed.

Online Teaching Suite

The Online Teaching Suite is the teacher's version of the Interactive Textbook. Much more than a 'Teacher Edition', the Online Teaching Suite features the following:

- The ability to view students' working and give feedback – When a student has submitted their work online for an exercise, the teacher can view the student's work and can give feedback on each question.
- For Years 5 and 6, access to Chapter tests, Blackline Masters, Challenge exercises, curriculum support material, and more.
- For Years 7 to 10, access to Pre-tests, Chapter tests, Skillsheets, Homework sheets, curriculum support material, and more.
- A Learning Management System that combines task-management tools, a powerful test generator, and comprehensive student and whole-class reporting tools.

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CHAPTER

1

Number and Algebra

Consumer arithmetic

This chapter reviews some important practical financial topics, such as investing and borrowing money, income tax and GST, inflation, depreciation, profits and losses, discounts and commissions. Formulas for compound interest and depreciation are introduced.

Everything in this chapter requires calculations with percentages. We are assuming that you are using a calculator, so we have made little attempt to set questions where the numbers work out nicely.

Nevertheless, you should always look over your work and check that the answers to your calculations are reasonable and sensible.

When the calculator displays numbers with many decimal places, you will need to round the answer in some way that is appropriate in the context of the question. This is an important skill in everyday life.

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1A Review of percentages

We first review the calculation techniques involving percentages, which you have learned in previous years.

- To convert a percentage to a decimal, move the decimal point two places to the left. For example:

  $27\% = 0.27$

- To convert a percentage to a fraction, multiply by $\frac{1}{100}$. For example:

  $27\% = \frac{27}{100}$ and $2\frac{1}{2}\% = \frac{2\frac{1}{2}}{100} = \frac{5}{200} = \frac{1}{40}$

- To convert a decimal or a fraction to a percentage, multiply by 100%. For example:

  $0.35 = 0.35 \times 100\% = 35\%$ and $\frac{3}{5} = \frac{3}{5} \times 100\% = 60\%$

- To find a percentage of a quantity, convert the percentage to a decimal or a fraction, and then multiply the quantity by it. For example:

  3.5% of $1250 = 1250 \times 0.035 = 43.75$ or 3.5% of $1250 = 1250 \times \frac{35}{1000} = 43\frac{3}{4}$

- To calculate the percentage that one quantity, a, is of another quantity, b:
  - first convert both quantities to the same unit of measurement
  - then form the fraction $\frac{a}{b}$ and multiply it by 100%.

  For example, to express 32 cm as a percentage of 2.4 m:

  First, write 2.4 m = 240 cm

  Then $\frac{32}{240} \times \frac{100}{1}\% = 13\frac{1}{3}\%$

  So 32 cm is $13\frac{1}{3}\%$ of 2.4 m

Finding the original amount

We now introduce another important method that will be used with percentages throughout this chapter.

- To find the original amount, given 12% of it, divide by 12%.

Example 1

Ken saves 12% of his after-tax salary every week. If he saves \$108 a week, what is his after-tax salary?

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Solution

$$\text{Savings} = \text{after-tax salary} \times 12\%$$

Reversing this:

$$\begin{aligned}\text{After-tax salary} &= \text{savings} \div 12\% \\ &= \text{savings} \div 0.12 \qquad \text{(Replace 12\% by 0.12.)} \\ &= 108 \div 0.12 \\ &= \$900\end{aligned}$$

This technique of writing the percentage factor on the right and reversing the process using division is needed in many practical situations. It will be applied throughout this chapter to commissions, profit and loss, income tax and interest.

Commission

A **commission** is a fee that is charged by an agent who sells goods or services on behalf of someone else. The person who owns the goods or services is called the vendor, and the commission charged is usually determined as a percentage of the selling price.

Example 2

The Dandy Bay Gallery charges a commission of 8.6% on the selling price.

a An antique vase was sold recently for \$18 000. How much did the gallery receive, and how much was left for the vendor?

b The gallery received a commission of \$215 for selling a painting. What was the selling price of the painting, and what did the vendor actually receive?

Solution

a
$$\begin{aligned}\text{Commission} &= 18\,000 \times 8.6\% \\ &= 18\,000 \times 0.086 \\ &= \$1548\end{aligned}$$

$$\begin{aligned}\text{Amount received by vendor} &= 18\,000 - 1548 \\ &= \$16\,452\end{aligned}$$

b
$$\text{Commission} = \text{selling price} \times 8.6\%$$

Reversing this:

$$\begin{aligned}\text{Selling price} &= \text{commission} \div 8.6\% \\ &= 215 \div 0.086 \\ &= \$2500\end{aligned}$$

$$\begin{aligned}\text{Amount received by vendor} &= 2500 - 215 \\ &= \$2285\end{aligned}$$

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Profit and loss as percentages

Is an annual profit of $20 000 a great performance or a modest performance? For a business with annual sales of $100 000, such a profit would be considered very large. For a business with annual expenditure of $100 000 000, however, it would be considered a very poor performance.

For this reason, it is often convenient to express profit and loss as percentages of the total costs.

Example 3

The owners of Budget Shoe Shop spent $6 600 000 last year buying shoes and paying salaries and other expenses. They made a 2% profit on these costs.

a What was their profit last year?

b What was the total of their sales?

c In the previous year, their costs were $5 225 000 and their sales were only $5 145 000. What percentage loss did they make on their costs?

d Two years ago their costs were $5 230 000 and their sales were $6 125 000. What percentage profit did they make on their costs?

Solution

a
$$\begin{aligned}\text{Profit} &= 6\,600\,000 \times 2\% \\ &= 6\,600\,000 \times 0.02 \\ &= \$132\,000\end{aligned}$$

b
$$\begin{aligned}\text{Total sales} &= \text{total costs} + \text{profit} \\ &= 6\,600\,000 + 132\,000 \\ &= \$6\,732\,000\end{aligned}$$

c
$$\begin{aligned}\text{Last year, loss} &= \text{total costs} - \text{total sales} \\ &= 5\,225\,000 - 5\,145\,000 \\ &= \$80\,000\end{aligned}$$

$$\begin{aligned}\text{Percentage loss} &= \frac{80\,000}{5\,225\,000} \times \frac{100}{1}\% \\ &\approx 1.53\% \text{ (Correct to the nearest 0.01\%.)}\end{aligned}$$

$$\left[\begin{aligned}\text{Alternatively, profit} &= \text{total sales} - \text{total costs} \\ &= 5\,145\,000 - 5\,225\,000 \\ &= -\,\$80\,000 \\ \text{Percentage change} &\approx -1.53\% \\ &= 1.53\% \text{ loss}\end{aligned}\right]$$

d
$$\begin{aligned}\text{Profit} &= \text{total sales} - \text{total costs} \\ &= 6\,125\,000 - 5\,230\,000 \\ &= \$895\,000\end{aligned}$$

$$\begin{aligned}\text{Percentage profit} &= \frac{895\,000}{5\,230\,000} \times \frac{100}{1}\% \\ &\approx 17.11\% \text{ (Correct the nearest 0.01\%.)}\end{aligned}$$

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Example 4

Andrew's paint shop made a profit of 6.4% on total costs last year. If the actual profit was \$87 000, what were the total costs, and what were the total sales?

Solution

$$\text{Profit} = \text{costs} \times 6.4\%$$

$$\begin{aligned}\text{Reversing this, costs} &= \text{profit} \div 6.4\% \\ &= 87\,000 \div 0.064 \\ &= \$1\,359\,375\end{aligned}$$

$$\begin{aligned}\text{Hence, total sales} &= \text{profit} + \text{costs} \\ &= 87\,000 + 1\,359\,375 \\ &= \$1\,446\,375\end{aligned}$$

Income tax

Income tax rates are often **progressive**. This means that the more you earn, the higher the rate of tax you pay on each extra dollar earned.

Australian income tax rates are progressive, but they often change, so here is an example using the rates of the fictional nation of Plusionta, where taxation rates have not changed for many years.

Example 5

Income tax in the fictional nation of Plusionta is calculated as follows.

- There is no tax on the first \$12 000 that a person earns in any one year.
- From \$12 001 to \$30 000, the tax rate is 15c for each dollar over \$12 000.
- From \$30 001 to \$75 000, the tax rate is 25c for each dollar over \$30 000.
- For incomes exceeding \$75 000, the tax rate is 35c for each dollar over \$75 000.

Find the income tax payable by a person whose taxable income for the year is:

a \$10 500 **b** \$26 734 **c** \$72 000 **d** \$455 000

Solution

a There is no tax.

b Tax on first \$12 000 = \$0
Tax on remaining \$14 734
$= 14\,734 \times 0.15$
$= \$2210.10$
This is the total tax payable.

(continued over page)

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c Tax on first \$12 000 = \$0

Tax on next \$18 000
= 18 000 × 0.15
= \$2700

Tax on remaining \$42 000
= 42 000 × 0.25
= \$10 500

Total tax = 2700 + 10 500
= \$13 200

d Tax on first \$12 000 = \$0

Tax on next \$18 000 = \$2700

Tax on next \$45 000
= 45 000 × 0.25
= \$11 250

Tax on remaining \$380 000
= 380 000 × 0.35
= \$133 000

Total tax = 2700 + 11 250 + 133 000
= \$146 950

Simple interest

When money is lent by a bank or other lender, whoever borrows the money normally makes a payment, called **interest**, for the use of the money.

The amount of interest paid depends on:

- the **principal**, which is the amount of money borrowed
- the **rate** at which interest is charged
- the **time** for which the money is borrowed.

This section will deal only with **simple interest**. In simple interest transactions, interest is paid on only the original amount borrowed.

Conversely, if a person invests money in a bank or elsewhere, the bank pays the person interest because the bank uses the money to finance its own investments.

Formula for simple interest

Suppose that I borrow \$$P$ for T years at an interest rate of R per annum.

$$\begin{aligned}\text{Interest paid at the end of each year} &= P \times R \\ \text{Total interest, \$}I\text{, paid over } T \text{ years} &= P \times R \times T \\ &= PRT\end{aligned}$$

This gives us the well-known **simple interest formula**.

$$I = PRT \qquad (\text{Interest} = \text{principal} \times \text{rate} \times \text{time})$$

Note: The interest rate is normally given per year, so the time must also be written in years. In some books, R is written as $r\%$.

'Per annum' means 'per year'. It will sometimes be abbreviated to 'p.a.'.

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Example 6

Find the simple interest on $16 000 for eight years at 7.5% p.a.

Solution

$$\begin{aligned} I &= PRT \\ &= 16\,000 \times 7.5\% \times 8 \\ &= 16\,000 \times 0.075 \times 8 \\ &= 9600 \end{aligned}$$

Thus, the simple interest is $9600.

Reverse use of the simple interest formula

There are four pronumerals in the formula $I = PRT$. If any three are known, then substituting them into the simple interest equation allows the fourth to be found.

Example 7

John borrows $120 000 from his parents to put towards an apartment. His parents agree that John should only pay simple interest on what he borrows. Ten years later, John repays his parents $216 000, which includes simple interest on the loan. What was the interest rate?

Solution

$P = 120\,000$ and $T = 10$.

The total interest paid was $216 000 – $120 000 = $96 000, so $I = 96\,000$

$$\begin{aligned} I &= PRT \\ 96\,000 &= 120\,000 \times R \times 10 \\ R &= \frac{96\,000}{1\,200\,000} \times \frac{100}{1}\% \\ &= 8\% \end{aligned}$$

(Interest rates are normally written as percentages.)

The interest rate was 8%.

Simple interest formula

- Suppose that a principal $$P$ is invested for T years at an interest rate R p.a. Then the total interest $$I$ is given by:

  $I = PRT$
- If the interest rate R is given per year, the time T must be given in years.
- The formula has four pronumerals. If any three are known, the fourth can be found by substitution and solving the resulting equation.

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Exercise 1A

1 Express each percentage as a decimal.

a 56% **b** 8.2% **c** 12% **d** 3.75%

e 215% **f** 0.8% **g** $88\frac{1}{4}\%$ **h** $\frac{7}{8}\%$

2 Express each percentage as a fraction in lowest terms.

a 45% **b** 64% **c** $67\frac{1}{2}\%$ **d** $66\frac{2}{3}\%$

e 8.25% **f** 5.6% **g** 120% **h** 150%

i 7.25% **j** $12\frac{3}{4}\%$ **k** $\frac{1}{2}\%$ **l** 7.8%

3 Express each fraction or decimal as a percentage.

a $\frac{4}{5}$ **b** $\frac{7}{8}$ **c** $\frac{7}{16}$ **d** $1\frac{1}{2}$

e $\frac{9}{20}$ **f** $\frac{5}{3}$ **g** 0.46 **h** 0.025

i 1.4 **j** 1.125 **k** 0.000 75 **l** $2\frac{1}{4}$

4 Copy and complete this table.

	Percentage	Fraction	Decimal
a	64%		
b		$\frac{3}{5}$	
c			0.16
d	20.5%		
e			1.4
f		$\frac{5}{8}$	

5 Evaluate each amount, correct to two decimal places.

a 15% of 60 **b** 36% of 524

c 120% of 436 **d** 140.5% of 720

e 3.8% of 73 **f** 0.5% of 220

6 Evaluate each amount, correct to the nearest cent where necessary.

a 52% of \$50 **b** 24.2% of \$1050

c 110% of \$1590 **d** 0.30% of \$900

e $8\frac{1}{4}\%$ of \$2000 **f** $\frac{3}{4}\%$ of \$1060

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7 Find what percentage the first quantity is of the second quantity, correct to one decimal place.

a 9 km, 150 km

b \$5, \$400

c 28 kg, 600 kg

d 80 m, 50 m

8 Find what percentage the first quantity is of the second quantity, correct to two decimal places. You will first need to express both quantities in the same unit.

a 48 cents, \$10.00

b 3.4 cm, 2 m

c 28 hours, 4 weeks

d 250 m, 8 km

e 40 km, 1250 m

f 1 day, 2 years

9 There are 640 students at a primary school, 7% of whom have red hair. Calculate the number of students in the school who have red hair.

10 A sample of a certain alloy weighs 2.6 g.

a Aluminium makes up 58% of the alloy. What is the weight of the aluminium in the sample?

b The percentage of lead in the alloy is 0.28%. What is the weight of the lead in the sample?

11 A soccer match lasted 94 minutes (including injury time). If Team A was in possession for 65% of the match, for how many minutes and seconds was Team A in possession?

12 A football club with 15 000 members undertook a membership drive, and the membership increased by 110%.

a How many new members joined the club?

b What is the size of the club's membership now?

Example 1

13 Find the original quantity, given that:

a 5% of it is \$24

b 30% of it is 72 minutes

c 90% of it is 216 cm

d 7% of it is \$15.26

e 0.5% of it is 4 mm

f 15% of it is 56 mm

14 Sometimes customers pay a deposit on an item and then later pay the rest of the full price. Find the full price when a deposit of \$570 is 30% of the full price.

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Example 2 **15** Find the selling price if the commission and the commission rate are as given.

a Commission \$46, rate 8%

b Commission \$724, rate 5.6%

Example 3 **16** Find the percentage profit or loss on costs in these situations.

a Costs \$26 000 and sales \$52 000

b Costs \$182 000 and sales \$150 000

Example 4 **17** **a** A company made a profit of \$28 000, which was a 5.4% profit on its costs. Find the costs and the total sales.

b A company made a loss of \$750 000, which was a 6.5% loss on its costs. Find the costs and the total sales.

Example 5 **18** This question uses the income tax rates in the fictional nation of Plusionta. They are:

- There is no tax on the first \$12 000 that a person earns in any one year.
- From \$12 001 to \$30 000, the tax rate is 15c for each dollar over \$12 000.
- From \$30 001 to \$75 000, the tax rate is 25c for each dollar over \$30 000.
- For incomes exceeding \$75 000, the tax rate is 35c for each dollar over \$75 000.

a Find the income tax payable on:

i \$9000 **ii** \$15 000 **iii** \$38 000 **iv** \$400 000

b What percentage of each person's income was paid in income tax in parts **i–iv** of part **a**?

c Find the income if the income tax on it was:

i \$1580 **ii** \$3860 **iii** \$15 200 **iv** \$15 000

Example 6 **19** \$20 000 is invested at 8% p.a. simple interest for five years.

a How much interest will be earned each year?

b Use the formula $I = PRT$ to find how much interest will be earned over the five-year period.

20 Find the total simple interest earned in each investment.

a \$4000 for three years at 6% p.a.

b \$7500 for six years at 4.5% p.a.

Example 7 **21** Find the rate R in each simple interest investment.

a Interest of \$7200 on \$8000 for 12 years

b Interest of \$3 400 000 on \$12 500 000 for four years

22 Find the time T involved in each simple interest investment.

a Interest of \$2500 on \$1000 at 5% p.a. **b** Interest of \$91 200 on \$30 000 at 8% p.a.

23 Find the principal P in each simple interest investment.

a Interest of \$4320 at 4.8% p.a. for six years

b Interest of \$5020 at 6.75% p.a. for three years

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1B Percentage increase and decrease

When a quantity is increased or decreased, the change is often expressed as a percentage of the original amount.

This section reviews a concise method of dealing with percentage increase and decrease. The method will be applied in various ways throughout the remaining sections of the chapter.

Percentage increase

The Shining Path Cleaning Company made a profit of \$421 000 last year, and increased its profit this year by 23%.

We can find the new profit in one step by using the fact that the new profit is $100\% + 23\% = 123\%$ of the old profit.

$$\begin{aligned}\text{New profit} &= 421\,000 \times 123\% \\ &= 421\,000 \times 1.23 \\ &= \$517\,830\end{aligned}$$

When using a calculator, this is a simpler method than calculating the profit separately and adding it on. It will also allow us to handle repeated increases more easily and will make it simpler to reverse the process.

Percentage decrease

The same method can be used to calculate percentage decreases. For example, Grey Gully Station recently sold 41% of its 2288 head of cattle to the meatworks.

We can calculate how many head of cattle the station now has by using the fact that $100\% - 41\% = 59\%$ of its cattle remain.

$$\begin{aligned}\text{Number of remaining head of cattle} &= 2288 \times 59\% \\ &= 2288 \times 0.59 \\ &\approx 1350 \text{ (Correct to the nearest integer.)}\end{aligned}$$

Percentage increase and decrease

- To increase an amount by, say, 15%, multiply by $1 + 0.15 = 1.15$.
- To decrease an amount by, say, 15%, multiply by $1 - 0.15 = 0.85$.

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Finding the percentage increase or decrease

The method used in the following example is in keeping with the other methods covered in this chapter. It requires fewer calculations than finding the actual increase or decrease and then expressing that change as a percentage of the original amount. In all cases, subtracting the calculated percentage by 100% determines the percentage change.

Example 8

The water stored in the main Warrabimbie Dam has increased from 1677 gigalitres to 2043 gigalitres in three months. What percentage increase is this?

Solution

$$\frac{\text{New storage}}{\text{Old storage}} = \frac{2043}{1677} \times \frac{100}{1}\%$$

$$\approx 121.82\% \text{ (Correct to the nearest 0.01\%.)}$$

Thus, the storage has increased by about $121.82\% - 100\% = 21.82\%$.

Reversing the process to find the original amount

Harry claims that his mathematics mark of 78 constitutes a 45% increase on his previous mathematics mark. What was his previous mark?

This mark is $100\% + 45\% = 145\%$ of the previous mark.

Hence, this mark $=$ previous mark $\times$ 1.45

$$\begin{aligned}\text{Reversing this, previous mark} &= \text{this mark} \div 1.45\\ &= 78 \div 1.45\\ &\approx 54 \text{ (Correct to the nearest mark.)}\end{aligned}$$

Thus, to find the original amount, we divide by 1.45, because dividing by 1.45 is the reverse process of multiplying by 1.45.

Exactly the same principle applies when an amount has been decreased by a percentage, as shown in the following example.

Example 9

The price of bananas has decreased by 70% over the last year to $3 per kilogram. What was the price a year ago?

Solution

The new price is $100\% - 70\% = 30\%$ of the old price.

Hence, new price $=$ old price $\times$ 0.30

$$\begin{aligned}\text{Reversing this, old price} &= \text{new price} \div 0.30\\ &= 3.00 \div 0.3\\ &= \$10 \text{ per kilogram}\end{aligned}$$

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Example 10

Ria has had mixed results with the shares that she bought three years ago. Shares in White Manufacturing rose 37% to \$14.56, but shares in Black Tile Distributors fell 28% to \$8.76. Find the prices she originally paid for these two shares, correct to the nearest cent.

Solution

White Manufacturing shares are now $100\% + 37\% = 137\%$ of their previous value.

Thus, new value = original price × 1.37

$$\begin{aligned}\text{Reversing this, original price} &= 14.56 \div 1.37\\ &\approx \$10.63 \text{ (Correct to the nearest cent.)}\end{aligned}$$

Black Tile Distributors shares are now $100\% - 28\% = 72\%$ of their original value.

Thus, new value = original price × 0.72

$$\begin{aligned}\text{Reversing this, original price} &= 8.76 \div 0.72\\ &\approx \$12.17 \text{ (Correct to the nearest cent.)}\end{aligned}$$

Finding the original amount

- To find the original amount after an increase of, say, 15%, divide the new amount by $1 + 0.15 = 1.15$.
- To find the original amount after a decrease of, say, 15%, divide the new amount by $1 - 0.15 = 0.85$.

Discounts

It is common for a shop to **discount** the price of an item. This can be done to sell stock of a slow-moving item more quickly, or simply to attract customers into the shop.

Discounts are normally expressed as a percentage of the original price.

Example 11

The Tie Knot Shop is expecting new stock and needs to make room on its shelves. It has discounted all its prices by 45% to try to sell some of its existing stock.

a What is the discounted price of a tie with an original price of \$90?

b What was the original price of a tie with a discounted price of \$90?

Solution

The discounted price of each item is $100\% - 45\% = 55\%$ of the old price.

a
$$\begin{aligned}\text{Discounted price} &= \text{original price} \times 0.55\\ &= 90 \times 0.55\\ &= \$49.50\end{aligned}$$

b
$$\begin{aligned}\text{Original price} &= \text{discounted price} \div 0.55\\ &= 90 \div 0.55\\ &\approx \$163.64\end{aligned}$$
(Correct to the nearest cent.)

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The GST

In 1999 the Australian Government introduced a Goods and Services Tax, or GST for short. This tax applies to nearly all goods and services in Australia.

The current rate of GST is 10% of the pre-tax price of the good or service.

- When GST applies, GST is added to the pre-tax price. This is easily done by multiplying by 1.10.
- Conversely, if a quoted price already includes the GST, the pre-tax price is obtained by dividing by 1.10.

Example 12

The current GST rate is 10% of the pre-tax price.

a The pre-tax price of a large fridge is $2150. What will the fridge cost after GST is added, and how much will be paid to the government?

b I recently paid $495 to have a tree pruned. What was the price before adding GST, and how much GST was paid to the government?

Solution

The after-tax price is 110% of the pre-tax price.

a
$$\begin{aligned}\text{After-tax price} &= 2150 \times 1.10 \\ &= \$2365\end{aligned}$$
$$\begin{aligned}\text{Tax} &= 2365 - 2150 \\ &= \$215\end{aligned}$$

Alternatively,
$$\begin{aligned}\text{tax} &= 2150 \times 0.1 \\ &= 215\end{aligned}$$
$$\begin{aligned}\text{After-tax price} &= \$2150 + \$215 \\ &= \$2365\end{aligned}$$

b
$$\begin{aligned}\text{Pre-tax price} &= 495 \div 1.10 \\ &= \$450\end{aligned}$$
(Divide by 1.10 to reverse the process.)
$$\begin{aligned}\text{Tax} &= 495 - 450 \\ &= \$45\end{aligned}$$

Inflation

The prices of goods and services in Australia and other countries usually increase by a small amount every year. This gradual rise in prices is called **inflation**, and is measured by taking the average percentage increase in the prices of a large range of goods and services.

Other things, such as salaries and pensions, are often adjusted automatically every year to take account of inflation.

High rates of inflation are damaging to society, and governments generally try to keep inflation low.

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Example 13

The economy in Espirito Santo is booming as a result of its mineral exports, but unfortunately, with a change of government, inflation has also taken hold. Last year inflation was 28%, meaning that on average, prices have increased by 28% over the last year.

a If the average winter electricity bill was \$460 last year, give an estimate of this year's bill, based on the inflation rate.

b If a new Hunter Flash station wagon now costs \$38 000, give an estimate of its cost a year ago, based on the inflation rate, correct to the nearest \$100.

Solution

We estimate this year's prices as $100\% + 28\% = 128\%$ of last year's prices.

a $\text{Estimate of this year's bill} = 460 \times 1.28$

$\approx \$588.80$

b $\text{Estimate of cost last year} = 38\,000 \div 1.28$

$\approx \$29\,700$ (Correct to the nearest \$100.)

Exercise 1B

1 Increase each amount by the given percentage.

a \$570, 10% **b** \$9320, 5% **c** \$456, 6% **d** \$3120, 8%

2 Decrease each amount by the given percentage.

a \$9000, 10% **b** \$4560, 5% **c** \$826, 3% **d** \$9520, 4%

3 Traffic on all roads has increased by an average of 12% during the past 12 months. By multiplying by $112\% = 1.12$, estimate the number of vehicles now on a road where the number of vehicles a year ago was:

a 32 000 per day **b** 153 000 per day **c** 248 per day

4 Rainfall across Victoria has decreased over the last 10 years by about 38%. By multiplying by $62\% = 0.62$, estimate, correct to the nearest mm, the annual rainfall this year at a place where the rainfall 10 years ago was:

a 700 mm **b** 142 mm **c** 1268 mm

Example 8

5 The number of shops in different shopping centres in Borrington changed from 2011 to 2012, but by quite different percentage amounts. Find the percentage increase or decrease in the number of shops where the numbers during 2011 and 2012, respectively, were:

a 200 and 212 **b** 85 and 160 **c** 156 and 122 **d** 198 and 110

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Example 9

6 **a** An amount is decreased by 10% and the new amount is \$567. What was the original amount?

b An amount is increased by 10% and the new amount is \$5676. What was the original amount?

7 Phoenix Finance Pty Ltd recently issued bonus shares that increased by 14% the number of shares held by each of the company's shareholders. By dividing by 114% = 1.14, find the original holding of a shareholder who now holds:

a 228 shares **b** 8321 shares **c** 77 682 shares

Example 10

8 A research institute is trying to find out how much water Lake William had in it 8000 years ago. The lake now contains 7600 megalitres, but there are various conflicting theories about the percentage change over the last 8000 years. Find how much water the lake had 8000 years ago, correct to the nearest 10 megalitres, if in the last 8000 years the volume has:

a risen by 60% **b** fallen by 33% **c** risen by 312% **d** fallen by 88%

Example 11

9 A clothing store is offering a 35% discount on all its summer stock. Find the discounted price of an item with a marked price of:

a \$80 **b** \$48 **c** \$680 **d** \$1.60

Example 11

10 A furniture shop is offering a 55% discount at its end-of-year sale. Find the original price of an item with a discounted price of:

a \$1400 **b** \$327 **c** \$24.50

11 Mr Brown bought parcels of shares in June last year. He has a spreadsheet showing the value at which he bought his shares, the value at 31 December last year, and the percentage increase or decrease in their value. (Decreases are shown with a negative sign.) Unfortunately, a virus has corrupted one entry in each row of his spreadsheet. Help him fix his spreadsheet by calculating the missing values, correct to two decimal places.

Company	Value at purchase	Value at 31 December	Percentage increase
a	\$20 000		40%
b	\$14 268		–58%
c	\$3128.72		341.27%
d		\$80 000	15%
e		\$114 262	258.3%
f		\$32 516.24	–92.29%
g	\$50 000	\$52 000	
h	\$21 625	\$34 648	
i	\$48 372.11	\$40 072.11	

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