QUANTITATIVE TECHNIQUES IN MANAGEMENT

5th Edition

Mc Graw Hill Education

N D VOHRA

Quantitative Techniques in Management

Fifth Edition

About the Author

Dr N. D. Vohra has recently relinquished his tenure as Associate Professor in the Department of Commerce at Ramjas College, University of Delhi. Currently, he is teaching quantitative techniques to the Global Business Operations classes at Shri Ram College of Commerce, University of Delhi.

He obtained his postgraduate degree in commerce and doctorate in finance from Delhi School of Economics. His areas of interest are quantitative analysis, including statistical analysis, and finance. He has been teaching since 1974 and has an extensive experience including that at the Department of Commerce, Delhi School of Economics. He also taught at the Institute of Chartered Accountants of India and other institutions offering professional courses in management. Earlier, he served at Foundation of Technical Institutes, Iraq, and as Director of Research with the BLB Group of companies. He served at Geaddu College of Business Studies, Bhutan, in 2011.

He is a prolific writer and his books include *Business Statistics* and *Futures and Options* besides *Quantitative Techniques in Management*.

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McGraw Hill Education (India) Private Limited
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Chennai New York St Louis San Francisco Auckland Bogotá Caracas
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San Juan Santiago Singapore Sydney Tokyo Toronto

McGraw Hill Education (India) Private Limited

Published by McGraw Hill Education (India) Private Limited
444/1, Sri Ekambara Naicker Industrial Estate, Alapakkam, Porur, Chennai - 600 116

Quantitative Techniques in Management

Print Edition
ISBN-13: 978-93-5260-626-9
ISBN-10: 93-5260-626-4

E-book Edition
ISBN-13: 978-93-5260-627-6
ISBN-10: 93-5260-627-2

Managing Director: *Kaushik Bellani*

Portfolio Lead—B&E: *Nikhil Wadhera*
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Manager—Production: *Reji Kumar*

Typeset at Tej Composers, WZ 391, Madipur, New Delhi 110 063 and printed at

Cover Printed at:

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Dedicated
to the memory of
my parents

Preface to the Fifth Edition

It is a matter of great pleasure for me to present the readers with the fifth edition of *Quantitative Techniques in Management* that has been one of the most widely used books in the field of quantitative analysis for more than 25 years. While this edition continues to retain the clear and crisp pedagogy of the previous four editions, several significant changes have been made to enhance the content, quality and utility of the book in an endeavour to make it more useful to the students, teachers and other users of the subject.

The chapters in the new edition have been reorganised. The book is broadly divided into three sections. After the introductory chapter, the first section comprises chapters 2 through 9 and covers linear programming and related topics. The next section, consisting of six chapters,discusses more operations research techniques. The last four chapters constitute the third section and cover various statistical tools and concepts of investment analysis.

Ample changes have been made in several chapters for a greater logical flow and better understanding of the subject matter.

New to the Fifth Edition

- Introduction of MS Excel as means to solve various problems
- Solutions to linear programming and other optimization problems explained using Solver
- Comprehensive discussion on sensitivity analysis
- A new, Chapter-end feature, *QT in Practice* – which includes relatively large-sized problems at several chapter-ends.
- Many new problems including latest examination questions, both solved and unsolved
- Explanation of important concepts through boxed item

Online Supplements

We have made ancillary material available to the students and the instructors on our online learning center (OLC) as www.<to-be-provided>.com.

- **For Students**
  - Chapter key points
  - Detailed solutions to selected chapter-end practical problems
  - Multiple choice questions
  - Appendixes to Chapter 11 and 13
  - Statistical and Other Tables
  - Mathematics and Statistics Refresher including Matrix Algebra, Differential Calculus, Probability and Probability Distributions

- **For Instructors**
  - Detailed solutions to chapter-end practical problems
  - 300+ PowerPoint presentation slides
  - Multiple-Choice Questions with answers
  - Additional *QT in Practice* problems with solutions

Besides, learning objectives have been introduced in each chapter and the text has been organised according to these LOs. Also, chapter-end practical problems have been reorganised and tagged as per the particular learning objective/s. Many new problems including latest examination questions, both solved and unsolved, have been added, while some others have been replaced. Further, the chapter summary given in the previous edition chapters has been done away with. Instead, now the text is interspersed with boxes containing important points.

A noteworthy change made in this edition is the introduction of MS Excel as a means to solve various problems using Quantitative Techniques. Solutions to linear programming and related problems are explained using solver. Besides this, sensitivity analysis is also considered. Another significant change is the inclusion of relatively large-sized problems at several chapter-ends under the caption *QT in Practice*. These will help the students to apply particular techniques learnt to somewhat realistic situations.

Acknowledgements

This edition of the book owes its existence to several people who have contributed during its preparation. Dr Surbhi Singhal, my colleague at Ramjas College, has helped a lot in shaping the content on the use of MS Excel to solve QT problems. She thoroughly reviewed the material and made useful observations and suggestions to improve. I am deeply thankful to her for her efforts. I also wish to place on record all the help provided by my student, Ms Sarika Rakhyani, in many ways especially in weeding out the printing errors in the book. She also made candid observations and provided useful inputs. My heartfelt thanks are due to her.

I am also thankful to Dr Gurcharan Sachdeva, Assistant Professor at PGDAV College, for providing me assistance during the course of preparing this edition. My friends, Prof S.C. Makani and Prof D.K. Vaid have been constant source of encouragement to me. I am grateful to them.

Several mails from teachers and students all over the country have also contributed to make improvements in the book. I feel grateful to them all. Also, my thanks are due to the entire team at McGraw-Hill Education,India, who brought out this edition in a very industrious and professional manner.

I look forward to comments and observations from the readers of the book. These can be mailed to me at *ndvohra@hotmail.com* or *ndvohra@gmail.com*.

Dr N. D. Vohra

Publisher's Note

McGraw Hill Education (India) invites suggestions and comments from you, all of which can be sent to *info.india@mheducation.com* (kindly mention the title and author name in the subject line).

Piracy-related issues may also be reported.

Preface to the First Edition

Decision-making is a fundamental part of the management process and it pervades the activities of every business manager. In fact it is the manager's competence as a decision-maker that enables us to distinguish between a good manager and a bad one. Modern management is adopting and applying quantitative techniques to aid the process of decision-making in an ever increasing measure. This is due to the fact that an intelligent application of the appropriate tools can reduce an otherwise unwieldy and complex problem to one of manageable dimensions. Of course, the quantitative techniques are expected only to supplement—and not to supplant—the manager's sense of decision-making. He can use analytical tools in a wise manner only if he comprehends fully the underlying assumptions: what the analysis achieves, what compromises the model used makes with reality, and, above all, how the conclusions derived are to be adapted to the dynamic environmental conditions. Doubtless, however, a knowledge of quantitative analysis is a boon to the manager. Besides, the inclusion of operations research/quantitative techniques in management decision-making as an integral constituent of the management curricula of nearly all universities and professional institutions, such as the Institute of Chartered Accountants of India and the Institute of Cost and Works Accountants of India, is a pointer to the great significance that this field of study has assumed in the recent past.

In the present book, a modest attempt has been made to discuss some of the commonly used quantitative techniques in a wide spectrum of decision-making situations. It is addressed alike to the managers of today and tomorrow—the students pursuing post-graduate courses in commerce and management, the professional courses like CA and ICWA etc. The book is intended to provide a comprehensive presentation of the essential topics of quantitative decision-making. Towards this end, an attempt has been made to present the subject matter in a lucid style. It is aided by the inclusion of a large number of flow charts to display the schematics of the various algorithms. They have been included because they not only help to develop or extend skills that are fundamental to courses in computer science and management information systems, but also obviate the provision of algorithm summaries. An attempt has been made to develop various topics without resorting to rigorous mathematical sophistication but at the same time without an undue sacrifice in pedagogical foundation. Recondite notation and obscure manipulations have been scrupulously avoided. Wherever necessary, mathematical derivations have been relegated to the appendices. With a view to making the book self-contained, a review of the basic mathematics and statistics, required to understand the content of the book, has been provided. The book presents the application of various techniques through a large number of examples, review illustrations and exercises. A number of problems from various examinations have also been incorporated, Answers to the exercises with an asterisk (*) have been provided in Appendix A to the book.

The Instructors' Manual, containing detailed solutions to the exercises with asterisk mark and a few others, is also available.

Readers' views, observations, constructive criticism and suggestions are welcome.

Dr N. D. Vohra

Contents

1 Decision-Making and Quantitative Techniques

Chapter Overview

Decision-making is an integral part of the process of management. It is the competence as a decision-maker that distinguishes a good manager from others. While traditionally, decision-making has been considered as an art, the growing complexity and competitive nature of environment in which managements have to operate now necessitate that decisions be made more systematically using as much quantitative information as possible, along with a consideration of qualitative factors. This book is about the use of quantitative techniques in managerial decision-making.

Broadly speaking, decision-making involves choosing a course of action from various available alternatives. The job of a manager, in the process of selecting from among available alternatives, is facilitated in a large measure by the application of appropriate quantitative techniques when, and to an extent, a problem is quantified. This approach to decision-making is known by several names like operations research, management science, quantitative analysis, so on.

The contents of this chapter will help a manager to understand questions like the following:

- What is operations research and how has it evolved? What are its characteristic features?
- What is the methodology used in operations research? In this context, what is problem formulation, model building, acquisition of input data, solution and interpretation of the results obtained, model validation and implementation of the solution?
- What are different types of models, and what is the use of mathematical models in operations research?
- What are different classifications of solutions–feasible and infeasible; optimal and non-optimal; and unique and multiple optimal solutions?
- What is sensitivity analysis?
- How quantitative analysis is an integral part of the modern computer-based information systems and how are quantitative tools used in each of the subsystems?

Thus, this introductory chapter gives some details about the decision-making process and an idea about the nature and methodology of the quantitative analysis. Finally, a plan of the book is presented which contains a brief account of the contents of each of the chapters to follow.

Learning Objectives

After reading this chapter, you should be able to:

LO 1 Identify the elements of a decision and various decision-making situations
LO 2 Know the role of quantitative analysis in decision-making
LO 3 Recall the historical development of operations research
LO 4 Explain the nature and characteristic features of operations research
LO 5 Understand the methodology of operations research
LO 6 Describe operations research as an integral part of computer-based information systems

1.1 INTRODUCTION

Decision-making is an essential part of the management process. Although authorities differ in their definitions of the basic functions of management, everybody agrees that one is not a manager unless he or she has some authority to plan, organise and control the activities of an enterprise and behaviour of others. Thus, decision-making pervades the activities of every business manager. Further, since to carry out the key managerial functions of planning, organising, directing and controlling, the management is engaged in a continuous process of decision-making pertaining to each of them, we can go to the extent of saying that management may be regarded as equivalent to decision-making.

Traditionally, decision-making has been considered purely as an art, a talent that is acquired over a period of time through experience. It has been considered so because a variety of individual styles can be observed in the handling and successful solving of similar managerial problems by different people in actual business. However, the environment in which the management has to operate nowadays is complex and fast changing. There is a greater need for supplementing the art of decision-making by systematic and scientific methods. A systematic approach to decision-making is necessary because today's business and the environment in which it functions are far more complex than in the past, and the cost of making errors may be too high. Most of the business decisions cannot be made simply on the basis of rule of thumb, using common sense and/or 'snap' judgement. Common sense may be misleading and snap judgements may have painful implications. For large businesses, a single wrong decision may not only be ruinous but may also have ramifications in national economy. As such, the present-day management cannot rely solely on a trial-and-error approach and the managers have to be more sophisticated. They should employ scientific methods to help them make proper choices. Thus, the decision-makers in the business world today must understand the scientific methodology of making decisions. This calls for (1) defining the problem in a clear manner, (2) collecting pertinent facts, (3) analysing the facts thoroughly and (4) deriving and implementing the solution.

1.2 QUANTITATIVE APPROACH TO DECISION-MAKING: OR/MS

LO 1 Identify the elements of a decision and various decision-making situations

Managerial decision-making is a process by which the management, when faced with a problem, chooses a specific course of action from a set of possible options. In making a decision, a business manager attempts to choose the most effective course of action in the given circumstances in attaining the goals of the organisation.

Classification of Decision Situations

The various types of decision-making situations that a manager might encounter can be listed as follows:

1. Decisions under *certainty,* where all facts are known fully and for sure, or under *uncertainty* where the event that would actually occur is not known but probabilities can be assigned to various possible occurrences.
2. Decisions for one time period only, called *static* decisions, or a sequence of interrelated decisions made either simultaneously or over several time periods, called *dynamic* decisions.
3. Decisions where the opponent is *nature* (digging an oil well, for example) or a *rational opponent* (for instance, setting the advertising strategy when the actions of competitors have to be considered).

These classes of decision-making situations are not mutually exclusive and a given situation would exhibit characteristics from each class. Stocking of an item for sale in a certain trade fair, for instance, illustrates a static decision-making situation where uncertainty exists and nature is the opponent.

The elements of any decision are:

(a) a decision-maker, who could be an individual, group, organisation or society;
(b) a set of possible actions that may be taken to solve the decision problem;
(c) a set of possible states that might occur;
(d) a set of consequences (pay-off) associated with various combinations of courses of action and the states that may occur and
(e) the relationship between pay-off and the values of the decision-maker.

In an actual decision-making situation, the definition and identification of alternatives, the states and the consequences are most difficult, albeit not the most crucial aspects of the decision problem.

> Decision-making is an all-pervasive feature of management. It is a process by which a manager, when faced with a problem, chooses a specific course of action from among a set of possible alternatives. Decision-making may involve decisions under certainty or uncertainty; under conditions that may be static or dynamic and against nature or some rational opponent. The elements of a decision include a decision-maker, a set of possible actions, a set of possible states that might occur, a set of pay-offs associated with various combinations of actions and states and the relationship between pay-offs and values of the decision-maker.

Decision-making and Quantitative Analysis

LO 2 Know the role of quantitative analysis in decision-making

In real life, some decision-making situations are simple while others are not. Complexities in decision situations arise due to several factors. These include the complicated manner of interaction of the economic, political, technological, environmental and competitive forces in society, the limited resources of an organisation, the values, risk attitudes and knowledge of the decision-makers and the like. For example, a company's decision to introduce a new product will be influenced by such considerations as market conditions, labour rates and availability, and investment requirements and availability of funds. The decision will be of multidimensional response, including the production methodology, cost and quality of the product, price, package design, and marketing and advertising strategy. The result of the decision would conceivably affect every segment of the organisation. The essential idea of the quantitative approach to decision-making is that if the factors that influence the decisions can be identified and quantified, it becomes easier to resolve the complexity of the tools of quantitative analysis. In fact, a large number of business problems have been given a quantitative representation with varying degrees of success and it has led to a general approach which is variedly designated as *operations research* (or *operational research), management science, systems analysis, decision analysis, decision science* and so on. Quantitative analysis is now extended to several areas of business operations and represents probably the most effective approach to handling of some types of decision problems.

A significant benefit of attaining some degree of proficiency with quantitative methods is exhibited in the way problems are perceived and formulated. A problem has to be well defined before it can be formulated into a well-structured framework for solution. This requires an orderly and organised way of thinking.

A systematic approach to managerial decision-making requires application of scientific methods. Such a systematic approach is provided by quantitative analysis/operations research. However, qualitative factors should also be given due consideration along with quantitative analysis of a problem.

Two observations may be made here. First, it should be understood clearly that, although quantitative analysis represents a scientific approach to decision-making, a decision by itself does not become a good and right decision for adoption merely because it is made within an orderly and mathematically precise framework. Quantification at best is an aid to business judgement and not its substitute. A certain degree of constructive skepticism is as desirable in considering a quantitative analysis of business decisions as it is in any other process of decision-making. Further, some allowances should be made for qualitative factors involving morale, motivation, leadership and so on, which cannot be ignored. But they should not be allowed to dominate to such an extent that the quantitative analysis seems to be an interesting, but worthless, academic exercise. In fact, the manager should seek some balance between quantitative and qualitative factors.

Secondly, it may be noted that various names for quantitative analysis—operations research, management science and so on—connote more or less the same general approach. We shall not attempt to discuss the differences among various labels, as it is prone to create more heat than light, but only state that the basic reason for so many titles is that the field is relatively new, and there is no consensus regarding which field of knowledge it includes. In this book, we use the terms *management science, operations research* and *quantitative analysis* interchangeably.

We shall now briefly discuss operations research—its historical development, nature and characteristics, and methodology.

1.2.1 Historical Development of Operations Research (OR)

LO 3 Recall the historical development of operations research

While it is difficult to mark the 'beginning' of the operations research/management science, the scientific approach to management can be traced back to the era of Industrial Revolution and even to periods before that. But operations research, as it exists today, was born during the second world war when the British military management called upon a group of scientists to examine the strategies and tactics of various military operations, with the intention of efficient allocation of scarce resources for the war effort. During this period, many new scientific and quantitative techniques were developed to assist military operations. The name *operational research* was derived directly from the context in which it was used—research activity on operational areas of the armed forces. British scientists spurred the American military management to similar research activities (where it came to be known as operations research). Among the investigations carried out by them were included the determination of (i) optimum convoy size to minimise losses from submarine attacks, (ii) the optimal way to deploy radar units in order to maximise potential coverage against possible enemy attacks and (iii) the invention of new flight patterns, and the determination of the correct colour of the aircraft in order to minimise the chance of detection by the submarines.

After the war, operations research was adopted by the industry and some of the techniques that had been applied to the complex problems of war were successfully transferred and assimilated for use in the industrial sector.

The dramatic development and refinement of the techniques of operations research and the advent of digital computers are the two prime factors that have contributed to the growth and application of OR in the post-war period. In the 1950s, OR was mainly used to handle management problems that were clear-cut, well-structured and repetitive in nature. Typically, they were of a tactical and operational nature, such as inventory control, resource allocation, scheduling of construction projects and so on. Since the 1960s, however, formal approaches have been increasingly adopted for the less well-structured planning problems as well. These problems are strategic in nature and are the ones that affect the future of the organisation. The development of corporate planning models and those relating to the financial aspects of the business, for example, are such types of problems. Thus, in the field of business and industry, operations research helps the management determine their tactical and strategic decisions more scientifically.

1.2.2 Nature and Characteristic Features of OR

LO 4 Explain the nature and characteristic features of operations research

In general terms, we can regard operations research as being the application of scientific methods to decision-making. Thus, operations research attempts to provide a systematic and rational approach to the fundamental problems involved in the control of systems by making decisions which, in a sense, achieve the best results considering all the information that can be profitably used. A classical definition of OR is given by Churchman et al., '… Operations Research is the application of scientific methods, techniques and tools to problems involving the operations of systems so as to provide those in control of operations with optimum solutions to the problems'.* Thus, it may be regarded as the scientific method employed for problem-solving and decision-making by the management.

The significant features of operations research are given below.

1. Decision-making Primarily, OR is addressed to managerial decision-making or problem-solving. A major premise of OR is that decision-making, irrespective of the situation involved, can be considered a general systematic process that consists of the following steps:

(a) Define the problem and establish the criterion which will be used. The criterion may be the maximisation of profits, utility and minimisation of costs and so on.

(b) Select the alternative courses of action for consideration.

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* Churchman, C.W., Ackoff, R.L. and Acnoff, *Introduction to Operations Research*, John Wiley & Sons New York, 1977, p.9.

(c) Determine the model to be used and the values of the parameters of the process.
(d) Evaluate the alternatives and choose the one that is optimal.

2. Scientific Approach OR employs scientific methods for the purpose of solving problems, and there is no place of whims and guesswork in it. It is a formalised process of reasoning and consists of the following steps:

(a) The problem to be analysed is defined clearly and the conditions for observations are determined.
(b) Observations are made under varying conditions to determine the behaviour of the system.
(c) On the basis of the observations, a hypothesis describing how the various factors involved are believed to interact and the best solution to the problem is formulated.
(d) To test the hypothesis, an experiment is designed and executed. Observations are made and measurements are recorded.
(e) Finally, the results of the experiments are analysed and the hypothesis is either accepted or rejected. If the hypothesis is accepted, the best solution to the problem is obtained.

3. Objective OR attempts to locate the best or optimal solution to the problem under consideration. For this purpose, it is necessary that a measure of effectiveness be defined, which is based on the goals of the organisation. This measure is then used as the basis to compare the alternative courses of action.

4. Inter-disciplinary Team Approach OR is inter-disciplinary in nature and requires a team approach to a solution to the problem. No single individual can have a thorough knowledge of the myriad aspects of operations research and how the problems may be addressed. Managerial problems have economic, physical, psychological, biological, sociological and engineering aspects. This requires a blend of people with expertise in the areas of mathematics, statistics, engineering, economics, management, computer science and so on. Of course, it is not always so. Some problem situations may be adequately handled even by one individual.

5. Digital Computer Use of a digital computer has become an integral part of the operations research approach to decision-making. The computer may be required due to the complexity of the model, volume of data required or the computations to be made. Many quantitative techniques are available in the form of 'canned' programmes.

1.2.3 Methodology of Operations Research

LO 5 Understand the methodology of operations research

The basic and dominant characteristic feature of operations research is that it employs mathematical representations or models to analyse problems. This distinctive approach represents an adaptation of the scientific methodology used by other physical sciences. The scientific method translates a real given problem into a mathematical representation, which is solved and re-transformed into the original context. The OR approach to problem-solving consists of the following steps:

1. Formulate the problem.
2. Determine the assumptions (model building) and formulate the problem in a mathematical framework.
3. Acquire the input data.
4. Solve the model formulated and interpret the results.
5. Validate the model.
6. Implement the solution obtained.

The features of operations research include managerial decision-making; application of scientific approach; lay downing of the objective; involving inter-disciplinary team approach and the use of computers. Its methodology comprises (a) formulation of the problem, (b) building of a suitable model, (c) collection of relevant information, (d) solution of the model and interpretation of the results, (e) validation of the model used, and finally (f) implementation of the solution obtained.

However, one step need not be completed fully before the next is started. In fact, one or more of the steps may be modified to some extent before final results are implemented. This would, of course, necessitate the subsequent steps being modified.

The steps are shown in Figure 1.1 and we now discuss each of these one by one.

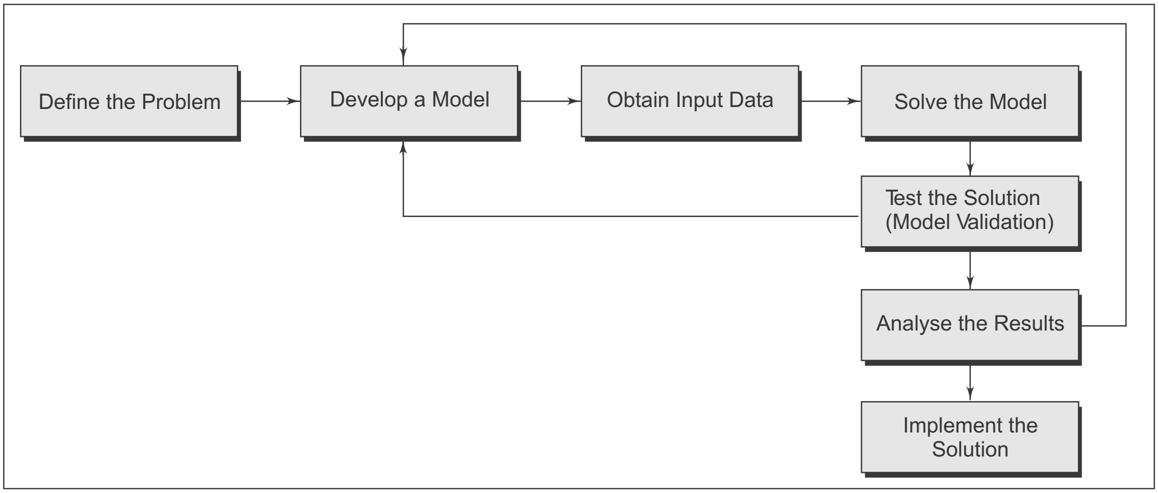

Figure 1.1 *Operations Research Methodology*

1. Problem Formulation The first step in quantitative analysis is to develop a clear and concise statement of the problem. In many cases, defining the problem proves to be the most important and difficult step. It is essential, therefore, that the root problem should be identified and understood in the first place. Logically speaking, we cannot expect to get the right answer if the problem is identified incorrectly. In that case, the solution derived from it is apt to be useless and all the efforts in that direction shall be a waste. The problem should be identified properly because often what is described as a problem may only be its symptom. For example, excessive costs *per se* do not constitute a problem. They are only an indication of some problems that may, for instance, be improper inventory levels, excessive wastage and the like.

Often the symptoms of a problem may extend beyond a single manager's control to other personnel and other departments in an organisation. It is essential, therefore, to go beyond symptoms of the problem and identify their actual causes. Also, one problem may be related to other problems and solving one problem without having regard to the others may make the matters worse. It is essential, therefore, that an analysis be made as to how the solution to one problem is likely to affect other problems or the situation in general. Further, it is likely for an organisation to have several problems. However, an analyst usually cannot deal with all the problems and has to focus only on a few of them. For most companies it translates in to selecting those problems whose solutions are likely to result in greatest profit increases or cost reductions. Thus, it is imperative that right problems be selected for solution. In sum, it is necessary for an analyst to understand that the formulation of a problem develops from a complicated interaction that involves the selection and interpretation of data between him and the management.

Once the problem has been identified, it is categorised as being standard or special. The *standard problems* are also known as *programmed problems.* As has already been mentioned, they are the well-structured problems characterised by routine, repetitive decisions that utilise specific decision-making techniques in their solution strategy. Standard solution procedures have been developed to handle such prototype problems. Examples of these problems include the assignment of workers to jobs, fixing the product-mix for a month and determination of the quantity of materials to be bought. On the other hand, there are *special* or *non-programmed* problems. They are unique and non-recurrent in nature and, therefore, ill-structured. Undertaking of a research and development project and the merger and consolidation decisions illustrate such types of decision situations.

2. Model Building Once the problem is defined, the next step is to build a suitable model. As already mentioned, the concepts of models and model building lie at the very heart of the operations research approach to problem-solving. A model is a theoretical abstraction of a real-life problem. In fact, many real-life situations tend to be very complex because there are literally innumerable inherent factors in any given situation. Thus, the decision-maker has to abstract from the empirical situation those factors which are most relevant to the problem. Having selected the critical factors, he combines them in some logical manners so that they form a counterpart or a model of the actual problem.

Thus, a model is a simplified representation of a real-world situation that, ideally, strips a natural phenomenon of its bewildering complexity and replicates its essential behaviour. Models may be represented in a variety of ways. They can be classified as physical and symbolic models.

(a) Physical Models

A physical model is a physical or schematic representation of the real thing. There are two types of physical models: iconic and analogue.

(i) Iconic Models They are essentially the scaled-up/down versions of the particular thing they represent. A model aeroplane in a wind tunnel, a model of a proposed building provided by an architect, models of the sun and its planets housed in a planetarium, a model of a particular molecular structure of a chemical are examples of iconic models, because they look like what they represent (except size). Maps, pictures or drawings may also be categorised as iconic models since they represent essentially the images of certain things.

The chief merit of an iconic model is that it is concrete and specific. It resembles visually the thing it represents and, therefore, there are likely to be fewer problems in translating any 'findings' from the model into the real-life situation. However, the disadvantage of such models is that they often do not lend themselves to manipulation for experimental purposes.

(ii) Analogue Models The analogue models use one set of properties to represent another set. To illustrate, an electrical network model may be used as an analogue model to study the flows in a transportation system. Similarly, a barometer that indicates changes in atmospheric pressure through movements of a needle is an example of analogue model and the contour lines on a map are analogues of elevation. In general, the analogue models are less specific and concrete but they are easier to manipulate as compared to the iconic models.

(b) Symbolic Models

Many real-life problems can be described by symbolic models or mathematical forms. These are the most general and abstract types of models. They employ letters, numbers and other types of symbols to represent the variables and their interrelationships. As such, they are capable of experimental manipulation most easily. The symbolic models can be verbal or mathematical. Whereas the verbal models describe a situation in spoken language or written words, the mathematical models employ mathematical notation to represent, in a precise manner, the variables of the real situations. The mathematical models take the form of mathematical relationships that portray the structure of what they are intended to represent. The use of a verbal versus mathematical model could be shown by the formula for finding the perimeter of a rectangle. A verbal model would express this problem as follows. The perimeter (P) of a rectangle is equal to the sum total of two times the length (L) and two times the width (W) of the rectangle. In contrast, the advantage of the mathematical model is demonstrated by the following statement: $P = 2L + 2W$. If applied to the same rectangle, both models would yield identical results. However, a mathematical model is more precise.

Symbolic models are used in operations research because they are easier to manipulate and they yield more accurate results under manipulation compared to the iconic or analogue models.

Use of Mathematical Models Various types of mathematical models are used in modern operations research. Two broad categories of these are deterministic and probabilistic models. A *deterministic model* is the one in which all parameters in the mathematical formulation are fixed at predetermined values so that no uncertainty exists. In a *probabilistic model***,** on the other hand, some or all the basic characteristics may be random variables (capable of assuming different values with given probabilities). In such models, uncertainty and errors are required to be given explicit consideration. Probabilistic models are also termed as stochastic or chance models.

The mathematical models comprise three basic components: result variables, decision variables and uncontrollable variables. The *result variables* reflect and measure the level of effectiveness of a system. They indicate how well a system performs. Also called as measures of performance or the objective, such variables are dependent variables, since they depend on the occurrence of decision and uncontrollable variables. The *decision variables* in turn are those where a choice can be made. These variables can be manipulated and are therefore controllable by the decision-maker. These variables are called independent variables. Finally, the *uncontrollable variables* refer to those factors in a decision situation which affect the result variables but are not in control of the decision-maker. These are also independent in nature since they affect the dependent variables. To illustrate, in the area of marketing, the decision variables may be the advertising budget, the number of regional salesmen employed, the number of products on sale and so on; the result variables may be total sales made, the market share for the company, level of customer satisfaction and so on while the uncontrollable variables may be the competitors' strategies, consumer incomes, consumer tastes and so on. Similarly, in the context of a production planning problem, the decision variables are the number of units of each of the products to produce; the result variable could be the total contribution or profit resulting while the availability of labour, raw materials, funds and market for each product exemplify the uncontrollable variables.

As mentioned earlier, the different components of a mathematical model are tied together with the relationships in the form of equations, inequalities and so on. Such a model consists of an objective function and describes how a dependent (result) variable is related to independent (decision) variables. For example, the profit function of a firm making two products can be stated as follows:

$$P = p_1x_1 + p_2x_2$$

in which P indicates the total profit of the firm, x_1 and x_2 are the number of units (independent) variables of the two products produced and sold, and p_1 and p_2 are the profit per unit on the two products, respectively (the uncontrollable variables).

The objective function is called for to maximise (or minimise), subject to certain constraints (representing the uncontrollable variables). For example, in this case of production, the firm might be able to sell no more than a certain number of units, say 80. Then the marketing constraint (an uncontrollable variable) can be expressed as follows:

$$x_1 + x_2 \leq 80$$

Similarly, other constraints, if any, of the system can be expressed.

> Models may be physical (including iconic and analogue) or symbolic (mathematical). Mathematical models, which may be deterministic or probabilistic, have three components: *decision variables* (which are controllable by the decision-maker); the *result variables* (those indicating effectiveness of the system and are not in control of the decision-maker) and the *uncontrollable variables* which generally emanate from the environment around the decision-maker. The decision variables and the uncontrollable variables are independent while the result variables are dependent in nature.

3. Obtaining Input Data Once an appropriate model has been formulated, the next step is to obtain the data to be used in the model as input. Since the quality of data determines the quality of output, the importance of obtaining accurate and complete data cannot be over-emphasised. Obviously, the finished product can be no better than the raw materials used. This situation may be described as GIGO: gold in, gold out or garbage in, garbage out.

Obtaining correct and relevant data may indeed be a difficult exercise when relatively large problems are involved. A number of sources, including company reports and documents, interviews with the company personnel and so on, may be used for collecting data.

4. Solution of Model Having formulated an appropriate model and collected the input data, the next stage in the analysis calls for the solution to the model and the interpretation of the solution in context of the given problem. A solution to a model implies determination of a specific set of decision variables that would yield a desired level of output. The desired level of output, in turn, is determined by the principle of choice adopted and represents the level which 'optimises'. Optimisation might mean maximising the level of goal attainment from a given set of resources or minimisation of cost as will satisfy the required level of goal attainment, or maximisation of ratio of the goal attainment to cost.

It may be noted that the solutions can be classified as being feasible or infeasible, optimal or non-optimal and unique or multiple.

(a) Feasible and Infeasible Solutions

A solution (a set of values of the decision variables, as already mentioned) which satisfies all the constraints of the problem is called a feasible solution, whereas an infeasible solution is the one which does not satisfy all the constraints. Since an infeasible solution fails to meet one or more of the system requirements, it is an unacceptable one. Only feasible solutions are of interest to the decision-maker.

(b) Optimal and Non-optimal Solutions

An optimal solution is one of the feasible solutions to a problem that optimises and is, therefore, the best among them. For example, for a multi-product firm working under some given constraints of capacity, marketing, finance and so on, the optimal solution would be that product-mix which would meet all the constraints and yield the maximum contribution margin towards profits. The feasible solutions other than the optimal solution are called non-optimal solutions. To continue with the example, several other product-mixes would satisfy the restrictions imposed and hence qualify for acceptance, but they would be ignored because lower contribution margins would be associated with them. They would be non-optimal.

(c) Unique and Multiple Solutions

If only one optimal solution to a given problem exists, it is called a unique solution. On the other hand, if two or more optimal solutions to a problem exist, which are equally efficient, then multiple optimal solutions are said to exist. Of course, these are preferable from the management's point of view as they provide a greater flexibility in implementation.

Once the principle of choice has been specified, the model is solved for optimal solution. For this, the feasible solutions are considered and of them the one (or more) that optimises is chosen. For this purpose, a complete enumeration may be made so that all the possible solutions are checked and evaluated. However, this approach is limited to those situations where the number of alternatives is small. Alternatively, and more commonly, methods involving algorithms may be used to get optimal solutions. It is significant to note that in contrast to complete enumeration, where all solutions are checked, an algorithm represents a trial-and-error process, where only a part of the feasible solutions are considered and the solutions are gradually improved until an optimal solution is obtained.

While algorithms exist for most of the standardised problems, there are also some numerical techniques, which yield solutions that are not necessarily optimal. Heuristics and simulation illustrate those methods. *Heuristics* are step-by-step logical *rules,* which, in a certain number of steps, yield some acceptable solution to a given problem. They are applied in those cases where no algorithms exist. Similarly, the technique of *simulation* is also applied where a given system is sought to be replicated and experimented with. Solutions using simulation need not be optimal because the technique is only descriptive in nature. Once a solution is obtained, it needs to be tested completely before it can be analysed and implemented. Because the solution depends on the input data and the model employed, both need testing. Such testing comprises determining the accuracy and completeness of the data used by a model and ensuring that the model used is logical and adequately represents the real situation.

Sensitivity Analysis In addition to the solution to the model formulated by any technique, sensitivity analysis or post-optimality analysis should also be performed. By sensitivity analysis we imply determination of the behaviour of the system to changes in the system inputs and specifications. Thus, it is *what if* analysis. This is done because the input data and the structural assumptions of the model may not be valid.

5. Model Validation The validation of a model requires determining whether the model can adequately and reliably predict the behaviour of the real system that it seeks to represent. Also, it involves testing the structural assumptions of the model to ascertain their validity. Usually, the validity of a model is tested by comparing its performance with the past data available in respect of the actual system. There is, of course, no assurance that the future performance of the system will continue to be in the same manner as in its past. Therefore, one must take cognisance of the changes in the system over time and adjust the model as required.

6. Implementation The final step is the implementation of the results. It is the process of incorporating the solution in the organisation. Implementation of solution is often more difficult than it may apparently seem. No standard prescription can be given, which would ensure that the solution obtained would automatically be adopted and implemented. This is because the techniques and models used in operations research may sound such and may be detailed in mathematical terms, but they generally do not consider the human aspects that are significant in implementation of solution. The impact of a decision may cut across various segments of the organisation, and the factors like resistance to change, desire to be consulted and informed, motivation and so on may come in the way of implementation. Equally important as the skill and expertise needed in developing a model is the requirement of tackling issues related to the factors which may have a bearing on the implementation of a solution in a given situation. Thus, a model, that secures a moderate theoretical benefit and is implemented, is better than a model which ranks very high on obtaining theoretical advantage but cannot be implemented. In fact, the importance of having managers in the organisation, who would act on the results of the study of a team that analyses the problem, can hardly be over-emphasised.

1.3 QUANTITATIVE ANALYSIS AND COMPUTER-BASED INFORMATION SYSTEMS

LO 6 Describe operations research as an integral part of computer-based information systems

Quantitative analysis/operations research has become an integral part of the modern computer-based information systems. A computer-based information system comprises:

Hardware:	input, CPU, storage and output
Software:	general operating software, general and specialised application software
Files:	tapes, disks, and documents
Procedures:	user, input and operating procedures
People:	managers, analysts, technical support personnel and operating personnel
Database:	information about various facets of the organisation

Such systems may include management information system (MIS), decision support system (DSS) and the use of artificial intelligence. Quantitative analysis tools are used in each of these subsystems.

1. Management Information Systems A management information system represents an organised way of managing information and data, which are vital organisational resources that are essential to the efficient and effective operations. Thus, it comprises a body of organised procedures for identification, collection, processing, retrieval and dissemination of information. It aims at providing right information to the right people in the right place at the right time. Provision of right information can often involve help of quantitative analysis. To illustrate, if a manager needs help to take ordering and inventory stocking decisions, forecasting models for projecting demand, inventory models to determine order quantities and so on are going to be quite helpful. To be able to have the needed information, it is necessary that the manager is able to interact with the computers 'on-line'. In the case of complexities, the quantitative analyst can aid the manager to retrieve the desired information from the system, by providing programmes to the manager for the same.

2. Decision Support Systems The advances in computer technology have witnessed development of decision support system (DSS) and artificial intelligence. In a decision support system, a system is developed to aid management in improving its decision-making. It supports, rather than replaces, managerial judgement. The presence of such a system often implies the use of computers to assist the managers in decision-making for semi-structured problems. The decision support system is interactive and allows the manager the use of *what if* questions, so that she may try different decisions, use modified data and witness results of such changes quickly. Its emphasis is on effectiveness of decision-making rather than on efficiency. Break-even analysis and profitability decisions, decision tables and expected values, decision trees, relevance trees, so on are all typical DSS models.

3. Artificial Intelligence and Expert Systems Although machine intelligence had been dreamt of for ages, the term *artificial* intelligence, to denote demonstration of intelligence by machines, evolved during the 1950s. Artificial intelligence includes the ability of computer systems and technology to think, see, learn, understand and use common sense—in other words, to mimic the human behaviour. It has grown into several important subsystems that have broad implications for quantitative analysis. Expert systems, an offshoot of the research on artificial intelligence, are information systems that attempt to support or automate decision-making and act like intelligent and rational decision-makers. They do this by storing and using knowledge about a specific, limited topic and produce conclusions based on the data that they receive.

Quantitative analysis is an integral part of the modern computer-based information systems including *management information systems* (MIS) representing an organised way of managing information and data; *decision support systems* (DSS) developed to aid management in improving in its decision-making and *artificial intelligence and expert systems* aiming to mimic human behaviour.

1.4 PLAN OF THE BOOK

As the title suggests, this book is an introduction to quantitative methods as an aid to managerial decision-making. Important quantitative techniques that may be employed by a manager in the evaluation of alternatives constitute the text of the book. The book is divided into three broad sections. The first section comprises Chapters 2–9 and considers programming problems. The second one consists of next six chapters and it discusses more operations research techniques. The last four chapters constitute the third section. This section covers various statistical tools and concepts of investment analysis. The chapter scheme of the book is given here.

Chapters 2–4 discuss ways and means of approaching the problems of resource allocation under the general category of *Linear Programming*. The attempt here is to seek an optimal solution to allocations where (i) there are a number of activities to be performed, (ii) there exist at least two different ways to perform these activities and (iii) resources are limited. The variables involved are assumed to be related linearly, and the boundaries or constraints of their interaction are also linear. Chapter 2 discusses the formulation of linear programming problems and the graphic approach to their solution, while Chapter 3 describes their solution using simplex algorithm. Chapter 4 is devoted to post-optimality analysis and the concept of duality.

Transportation and Transhipment Problems that deal with movement of a commodity from several origins to various destinations at minimum cost are discussed in Chapter 5. While the transportation problem allows goods to be shipped from various sources to different destinations, the transhipment permits goods being transhipped *en route* between sources as well. The *Assignment Problem*, discussed in Chapter 6, is one where certain items, may be people or activities, are to be assigned on a one-to-one basis to other items such as jobs or facilities, with the objective of minimising the cost or maximising the effectiveness of assignment.

Chapter 7 considers an extension of linear programming problems. It is addressed to *Integer Programming Problems* that restrict the decision variables to assume only integer values, and *Goal Programming Problems* that allow the manager to specify multiple goals instead of only a single goal of, say, profit maximisation. In this chapter, the problem of determining the optimal way of scheduling the tour for a *Travelling Salesman* is also considered.

Chapter 8 is titled *Theory of Games* and it deals with situations of conflict—where decisions are required to be made with the opponent being a rational decision-maker. The games can also be formulated as linear programming problems and solved as such. *Dynamic Programming*, an important quantitative tool to deal with a large number of problems, is described in Chapter 9. In this method, a problem is solved by dissecting it into a number of sub-problems. The solution to the sub-problems one-by-one eventually leads to the solution to the given problem. While dynamic programming is different from linear programming, many linear programming problems can also be solved using dynamic programming.

Chapter 10 considers the problem of *Sequencing* – the scheduling of different jobs to be performed on two or more machines. The next chapter, Chapter 11, deals with the *inventory* process and the question of how many units should be maintained in stock, including how often and how many units should be ordered, and the question of holding of safety stock. *Queuing Theory* is the subject of discussion in Chapter 12. Whenever people or objects reach a service station to receive some service, a waiting line is likely to form, particularly during the rush hours. Due to uncertainty of time and number of arrivals, there may a queue at certain times while the service facility may be idle at others. In general, larger the service facility the costlier would be operation but smaller will be the waiting lines; while a smaller facility has the opposite effect. The problem is to determine such a service level as will minimise the relevant costs.

The question of Replacement of Assets that wear out gradually and those that fail suddenly, as also the question of staff replacement are considered in Chapter 13. Chapter 14 addresses the tools of *Programme Evaluation and Review Technique* (PERT) and *Critical Path Method* (CPM), which are extremely useful for the purposes of planning and controlling the execution of complex projects. The next in sequence, Chapter 15 contains a discussion of *Markov Chains*, which deal basically with the problem of brand switching by customers of a product or a service. It discusses how the market shares are likely to be affected under conditions of customers shifting their preferences from one brand to another.

Chapter 16 introduces a powerful tool of quantitative analysis—the *Simulation*. The Monte Carlo analysis is introduced which is a technique of testing alternatives through trial and error approach. In effect, a system is sought to be 'copied', and through the technique of selecting random sequences of numbers from a probability distribution, we are able to test or simulate the outcomes associated with the alternatives that do not lend themselves to direct analysis and comparison. The next chapter, Chapter 17 reviews the decision process in the structured framework of *Decision Theory*. Several principles of decision-making are discussed in the chapter. Also included are the posterior analysis, decision tree and utility approaches to decision-making.

In Chapter 18, we consider *Investment Analysis and the Break-Even Analysis*. An attempt is made in this chapter to see how risk associated with an investment proposal may be quantified and incorporated into capital budgeting decisions. Some aspects relating to profit-volume relationship are also considered. Finally, the *Forecasting Techniques* are dealt with in Chapter 19. The forecasting techniques are used for making predictions of future demand or some other variable. They are helpful to the manager in as much as the manager is required to take decisions about the future which is at best uncertain.

TEST YOUR UNDERSTANDING

Mark the following statements as T (*True*) or F (*False*)

1. Decision-making is purely an art even in the modern age.
2. Decisions taken only on the basis of quantitative analysis can be sound and correct.