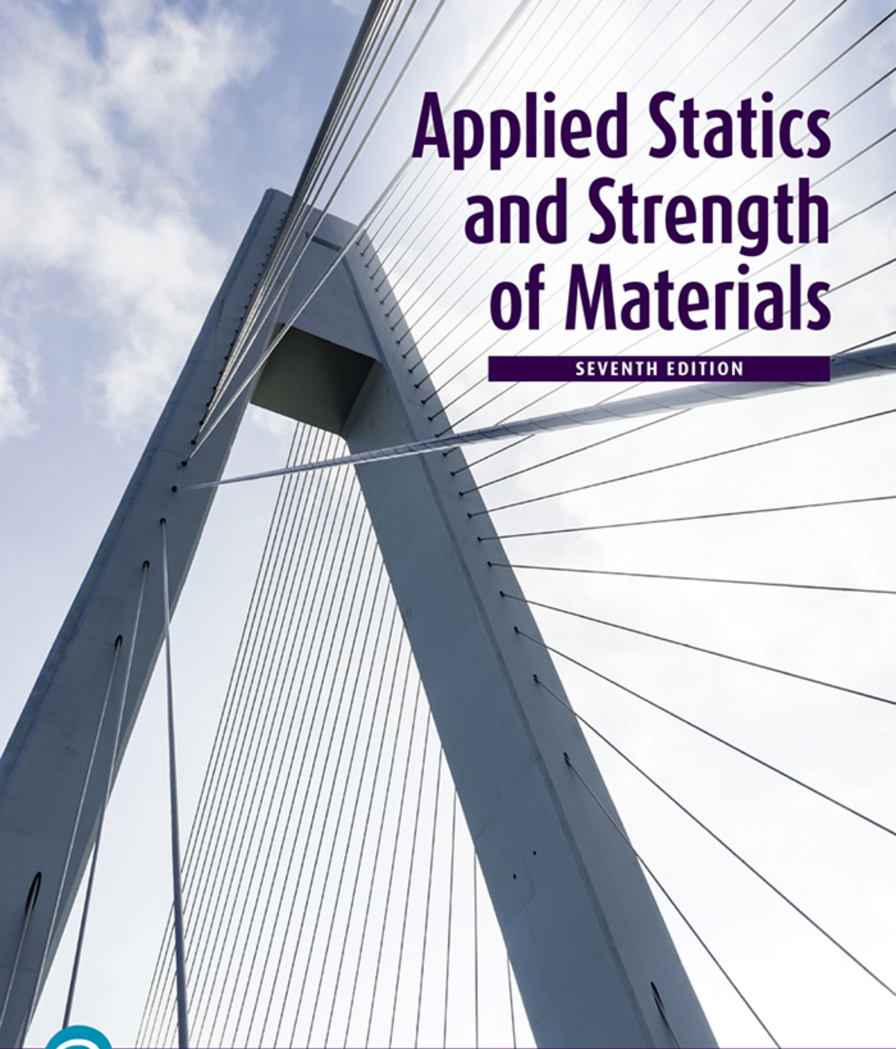
Applied Statics and Strength of Materials

SEVENTH EDITION



GEORGE F. LIMBRUNNER CRAIG T. D'ALLAIRD

APPLIED STATICS AND STRENGTH OF MATERIALS

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Seventh Edition

George F. Limbrunner, P.E. (Inactive)
Hudson Valley Community College (Emeritus)

Craig T. D'Allaird, P.E.
Hudson Valley Community College

Pearson

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BRIEF CONTENTS

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DETAILED CONTENTS

PREFACE

The seventh edition of *Applied Statics and Strength of Materials* has been completely updated and revised to meet current industry standards. It may be purchased or rented in two general formats – print and eText. The print version is now available as an affordable rent-to-own option for students. Several options for low cost digital eTexts are also available as a subscription or ownership. For access to Pearson eText, visit Pearson.com/learner.

As with previous editions, *Applied Statics and Strength of Materials* presents an elementary, analytical, and practical approach to the principles and physical concepts of statics and strength of materials. It is written at an appropriate mathematics level for engineering technology students, using algebra, trigonometry, and analytic geometry. An in-depth knowledge of calculus is not required for understanding the text or solving the problems.

This book is intended primarily for use in two-year or four-year technology programs in engineering, construction, or architecture. Much of the material has been classroom-tested in our Accreditation Board for Engineering and Technology (ABET) accredited engineering technology programs. The text can also serve as a concise reference guide for undergraduates in a first year engineering mechanics (statics) and/or strength of materials course in an engineering program. Although written primarily for technology students, this book can also be a valuable reference for those preparing for state licensing exams in engineering, architecture, or construction.

The book emphasizes mastery of basic principles, since it is this mastery that leads to successful solutions of real-world problems. This emphasis is achieved through numerous, step-by-step example problems, a logical and methodical presentation of material, and selection of topics geared toward student needs. This step-by-step approach to solving problems provides consistent and comprehensive solutions to problems that can be used as references. The principles and applications presented are applicable to many fields of engineering technology including civil, mechanical, construction, architectural, industrial, and manufacturing.

This seventh edition updates the content where necessary and rearranges and revises some of the material to enhance teaching aspects of the text. Some of the changes made to this text include:

- Re-wording of some chapters to modernize the text.
- Re-ordering of the end of chapter problems to eliminate the Supplemental Problems and to move the Computer Problems to the end of each chapter.
- Minor typographical errors as well as errors in the selected answers were corrected.
- All shape properties in Appendices A through D, Appendix I, and Appendix J have been updated to match the American Institute of Steel Construction (AISC) *Manual of Steel Construction*, 15th ed.

The book includes the following features:

- Each chapter is prefaced with learning objectives to emphasize the important concepts in the chapter.
- Problems at the end of each chapter are grouped and referenced to a specific section within the chapter. Problems are generally arranged in order of increasing difficulty.
- A summary at the end of each chapter provides a concise reference of the important concepts presented in that chapter.
- Tables of properties of areas and conversion factors for U.S. Customary/SI conversions are printed inside the covers for easy access.
- Most chapters contain computer problems following the section problems. These problems require students to develop computer programs to solve problems pertinent to the topics of the chapter. Any appropriate computer software may be used. The computer problems are another tool with which to reinforce students' understanding of concepts.
- Answers to selected problems are included at the back of the text.
- The primary unit system in this book remains the U.S. Customary System. SI, however, is fully integrated in both the text and the problems. Although full conversion to the metric system in the United States is not likely to happen soon, engineers and technicians must be fluent in both systems to participate in a global market.
- Design and analysis aids are furnished in the appendices, providing data in both U.S. Customary and SI units, to allow users to work though problems without additional references.
- Calculus-based proofs are presented in the appendices.
- The Instructor's Manual includes complete solutions for all the end-of-chapter problems in the text.

There is sufficient material in this book for two semesters of work in statics and strength of materials. In addition, by selecting certain chapters, topics, and problems, the instructor can adapt the book to other situations such as separate courses in statics (or mechanics) and strength of materials.

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ACKNOWLEDGMENTS

George Limbrunner was my professor, and later my mentor when I began teaching. Taking over this book from him is an honor and I cannot thank him enough.

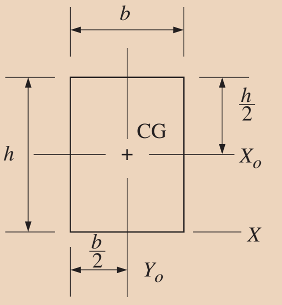
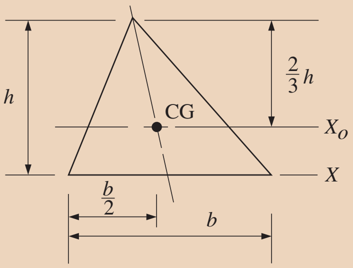
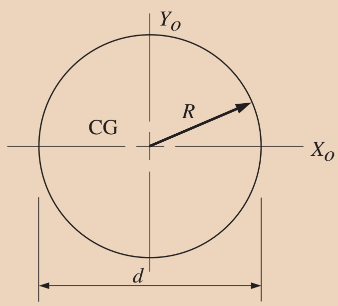
To my wife, Trisha and our three amazing children! I am blessed beyond words. All I can say is thank you so much for your support as I keep taking on these fun projects.

To my readers. Thank you for your continued emails, input, and corrections. No project is ever complete and I try to incorporate your comments when I can. Keep them coming.

Lastly, Hudson Valley Community College is a great place to work and learn, and I have to thank all the faculty who give their suggestions and input as I update the text. It is very appreciated.

Craig T. D'Allaird, P.E.

TABLE 3 Properties of areas

Shape	Area (A)	Moment of Inertia (I)	Radius of Gyration (r)	Polar Moment of Inertia (J)
 **Rectangle**	$A = bh$	$I_{x_o} = \frac{bh^3}{12}$ $I_{y_o} = \frac{hb^3}{12}$ $I_x = \frac{bh^3}{3}$	$r_{x_o} = \frac{h}{\sqrt{12}}$ $r_{y_o} = \frac{b}{\sqrt{12}}$ $r_x = \frac{h}{\sqrt{3}}$	$J_{CG} = \frac{bh}{12}(h^2 + b^2)$
 **Triangle**	$A = \frac{bh}{2}$	$I_{x_o} = \frac{bh^3}{36}$ $I_x = \frac{bh^3}{12}$	$r_{x_o} = \frac{h}{\sqrt{18}}$ $r_x = \frac{h}{\sqrt{6}}$	
 **Circle**	$A = \frac{\pi d^2}{4}$ $= 0.7854d^2$	$I_{x_o} = I_{y_o} = \frac{\pi d^4}{64}$ $= 0.04909d^4$ $= 0.7854R^4$	$r_{x_o} = r_{y_o} = \frac{d}{4}$	$J_{CG} = \frac{\pi d^4}{32}$

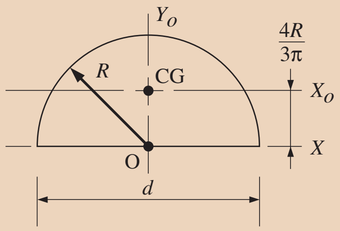
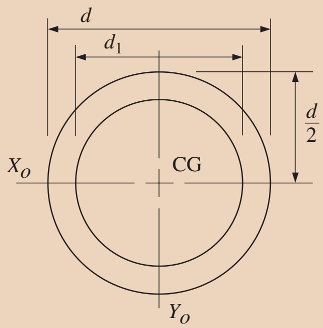
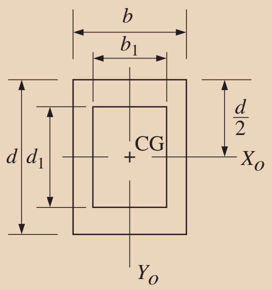
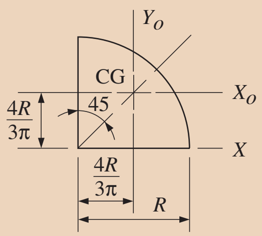
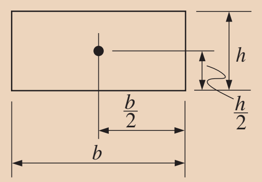
Shape	Area (A)	Moment of Inertia (I)	Radius of Gyration (r)	Polar Moment of Inertia (J)
 **Semicircle**	$A = \frac{\pi R^2}{2}$ $= 1.571R^2$	$I_{x_o} = 0.1098R^4$ $I_{y_o} = I_x = \frac{\pi R^4}{8}$ $= 0.3927R^4$	$r_{x_o} = 0.264R$ $r_{y_o} = r_x = \frac{R}{2}$	$J_{CG} = I_{x_o} + I_{y_o}$ $= 0.5025R^4$ $J_o = \frac{\pi R^4}{4}$
 **Hollow circle**	$A = \frac{\pi(d^2 - d_1^2)}{4}$ $= 0.7854(d^2 - d_1^2)$	$I_{x_o} = \frac{\pi(d^4 - d_1^4)}{64}$ $I_{y_o} = I_{x_o}$	$r_{x_o} = \frac{\sqrt{d^2 + d_1^2}}{4}$ $r_{y_o} = r_{x_o}$	$J_{CG} = \frac{\pi(d^4 - d_1^4)}{32}$
 **Hollow rectangle**	$A = bd - b_1 d_1$	$I_{x_o} = \frac{bd^3 - b_1 d_1^3}{12}$ $I_{y_o} = \frac{db^3 - d_1 b_1^3}{12}$	$r_{x_o} = \sqrt{\frac{bd^3 - b_1 d_1^3}{12A}}$ $r_{y_o} = \sqrt{\frac{db^3 - d_1 b_1^3}{12A}}$	$J_{CG} = I_{x_o} + I_{y_o}$
 **Quarter-circle**	$A = \frac{\pi R^2}{4}$	$I_{x_o} = I_{y_o} = 0.0549R^4$ $I_x = \frac{\pi R^4}{16}$	$r_{x_o} = r_{y_o} = 0.2644R$ $r_x = 0.5R$	$J_{CG} = 0.1098R^4$

TABLE 1 Conversion factors: U.S. Customary to SI units

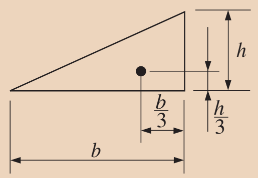
	Multiply		By	To Obtain
Length	inches	×	25.40	= millimeters
	feet	×	0.3048	= meters
	yards	×	0.9144	= meters
	miles (statute)	×	1.609	= kilometers
Area	square inches	×	645.2	= square millimeters
	square feet	×	0.0929	= square meters
	square yards	×	0.8361	= square meters
Volume	cubic inches	×	16,387.	= cubic millimeters
	cubic feet	×	0.028 32	= cubic meters
	cubic yards	×	0.7646	= cubic meters
	gallons (U.S. liquid)	×	0.003 785	= cubic meters
Force	pounds	×	4.448	= newtons
	kips	×	4448.	= newtons
Force per unit length	pounds per foot	×	14.594	= newtons per meter
	kips per foot	×	14,594.	= newtons per meter
Load per unit volume	pounds per cubic foot	×	0.157 14	= kilonewtons per cubic meter
Bending moment or torque	inch-pounds	×	0.1130	= newton meters
	foot-pounds	×	1.356	= newton meters
	inch-kips	×	113.0	= newton meters
	foot-kips	×	1356.	= newton meters
	inch-kips	×	0.1130	= kilonewton meters
	foot-kips	×	1.356	= kilonewton meters
Stress, pressure, loading (force per unit area)	pounds per square inch	×	6895.	= pascals
	pounds per square inch	×	6.895	= kilopascals
	pounds per square inch	×	0.006 895	= megapascals
	kips per square inch	×	6.895	= megapascals
	pounds per square foot	×	47.88	= pascals
	pounds per square foot	×	0.047 88	= kilopascals
	kips per square foot	×	47.88	= kilopascals
	kips per square foot	×	0.047 88	= megapascals
Mass	pounds	×	0.4536	= kilograms
Mass per unit volume (density)	pounds per cubic foot	×	16.02	= kilograms per cubic meter
	pounds per cubic yard	×	0.5933	= kilograms per cubic meter
Moment of inertia	inches^4	×	416,231.	= millimeters^4
Mass per unit length	pounds per foot	×	1.488	= kilograms per meter
Mass per unit area	pounds per square foot	×	4.882	= kilograms per square meter

TABLE 2 Areas and centroids of areas



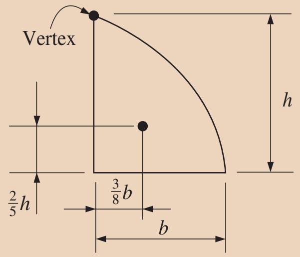
Rectangle

$A = bh$



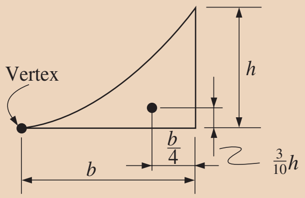
Right triangle

$A = \frac{1}{2}bh$



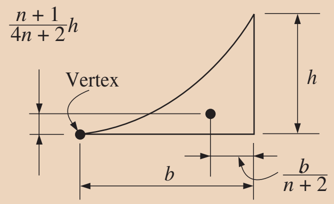
Second-degree parabola

$A = \frac{2}{3}bh$



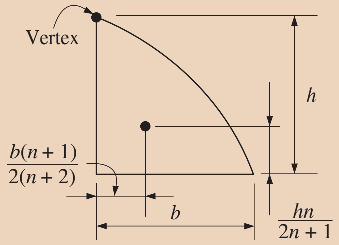
Second-degree parabola

$A = \frac{1}{3}bh$



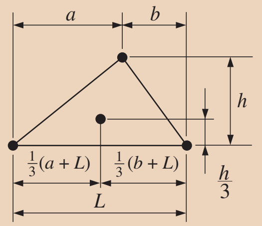
***n*th-degree parabola**

$A = \frac{bh}{n+1}$



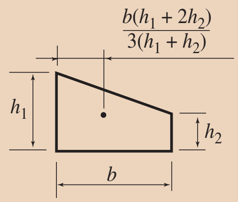
***n*th-degree parabola**

$A = \frac{nbh}{n+1}$



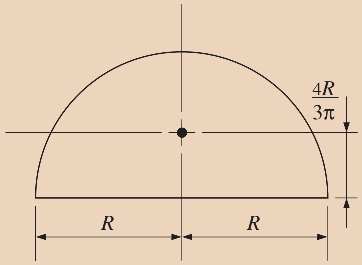
Triangle

$A = \frac{1}{2}Lh$



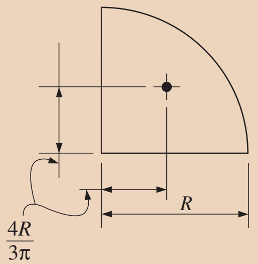
Trapezoid

$A = b\frac{h_1 + h_2}{2}$



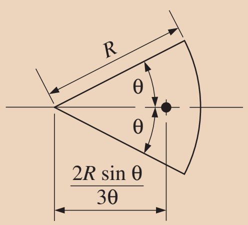
Semicircle

$A = \frac{\pi R^2}{2}$



Quarter-circle

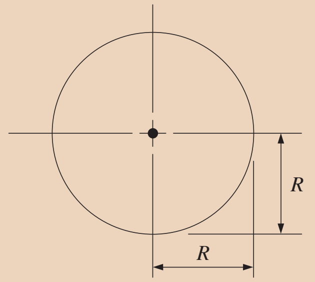
$A = \frac{\pi R^2}{4}$



Circular sector

$A = R^2\theta$

(Note: θ is in radians.)

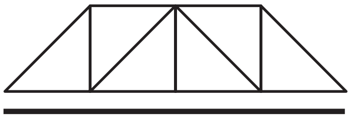


Circle

$A = \pi R^2$

CHAPTER ONE

INTRODUCTION



LEARNING OBJECTIVES

Upon completion of this chapter, readers will be able to:

- Develop an understanding of engineering mechanics as it applies to statics.
- Define methods for solving right and oblique triangles mathematically.
- Discuss and use significant figures for engineering purposes.
- Discuss the importance of units in calculations and discuss both U.S. Customary units and the SI unit system. Furthermore, readers should be able to convert between different units and perform dimensional analysis of equations.

1.1 MECHANICS OVERVIEW

Mechanics is the oldest of the physical sciences. Its laws and principles are the fundamentals of all branches of engineering. Like mathematics, mechanics should be thought of as a means to more advanced studies, such as design and analysis as well as research and development of buildings, bridges, automobiles, aircraft, spacecraft, and the like. Mechanics is the core of all these endeavors, and requires study by all those pursuing an education in engineering, architecture, or construction.

Mechanics deals essentially with the study of forces and their effects on both fluid and solid bodies that are at rest or in motion. A body at rest is termed *static*, while a body in motion is referred to as *dynamic*. As Figure 1.1 shows, this text will deal solely with the study of *statics*, or the effect of forces on solid, rigid bodies that are at rest.

In the study of statics, it is assumed that all solid bodies (parts of the structure or machine being considered) are perfectly rigid and do not deform, even under large forces. Statics is basic to the understanding of how structural components and complex systems of buildings, bridges, machines, and equipment perform their function. The spectrum of applications ranges from the very simple (e.g., a person standing on a plank that spans a creek) to the highly complex (e.g., aircraft/spacecraft structural systems). Statics is discussed through the first eight chapters of this text.

The remainder of the text is concerned with the study of *strength of materials*, which may be described as a study of the relationships between external forces acting on solid bodies and the internal responses generated by these forces. Here, solid bodies are assumed to be deformable, not rigid. To visualize this, think of a rubber band being stretched to the point of breaking. The external force (you pulling on it) causes the rubber band to elongate and narrow, typically changing color slightly, until reaching a point where it will no longer stretch. It then either breaks, or the force is removed, and it returns to its original shape, or sometimes a slightly deformed shape remains. If we knew the material properties of the rubber band, the principles learned through strength of materials would allow us to determine the force that would cause the rubber band to deform and then the force that would result in the rubber band breaking. As one can imagine, this is critically important to the design of structures, whether it be a bridge, building, or machine part.

The rubber band is a simple example. The design of any object must consider (a) the types of loads that come upon the structure and its parts and (b) how large, in what form, and of what material should these parts be made so that they may sustain these loads safely and economically.

Some of the loads to be supported may be known at the outset; they may be prescribed by codes or specifications. Some loads may have to be assumed based on experimentation, engineering experience, and/or judgment. Whatever the case, the principles of statics are used to determine and describe the system of forces that acts on the structure as a whole and on each individual part of the structure.

When all the forces that act on a given part are known, their effect with respect to the physical integrity of the part (i.e, their effect in elongating, bending, twisting, compressing, or breaking it) still must be determined. The study of the relations between the forces that act on a body and the changes they produce in its size and form, or the tendency they have to break it, is the province of strength of materials.

Thus, the sequence of statics and strength of materials allows for a beginning understanding of the basic laws and

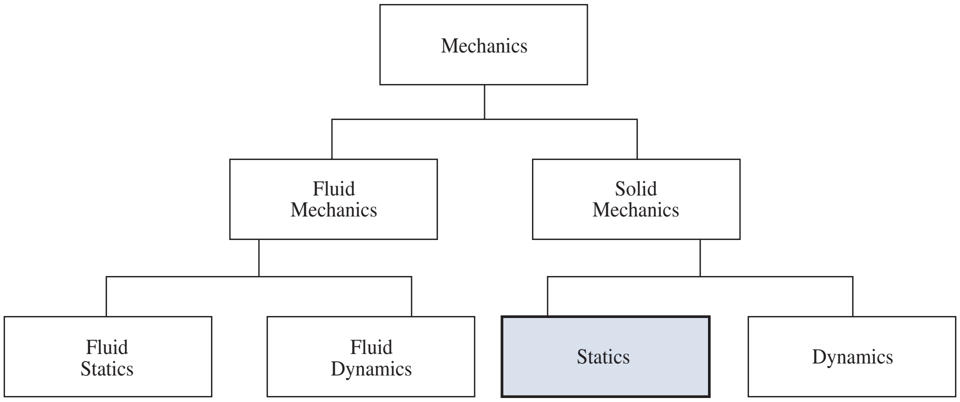


FIGURE 1.1 The field of mechanics.

principles involved in both the design and investigation of machine and structural elements.

1.2 APPLICATIONS OF STATICS

Perhaps some of us remember teeter-tottering with a parent or older sibling who, when sitting on their end, held us aloft at the high end of the plank (see Figure 1.2). Was there any thought, at the time, that we were at the mercy of the principles of statics? We quickly learned, however, to move the plank so that the pivot point would be farther away and more of the plank would be on our side. We couldn't explain why it worked, but it did, and we added that experience to our accumulated knowledge. What we actually did was apply one of the principles of statics.

Like math, we use statics on a daily basis without realizing it. Figure 1.3 illustrates some common applications of statics. All involve the analysis of forces and force systems. The basic understanding of forces in many structures and machines is intuitive, or perhaps based on experience, but a detailed analysis can be made only through the rigorous application of the principles of statics.

One such application is a simple floor system in residential construction. This appears straightforward enough at first glance. The principles of statics will make possible a detailed analysis of the magnitude of forces involved and how the forces pass from floor deck to supporting joists, to bearing beams, to posts, and eventually into the building's foundation. Another application of statics is found in the analysis of a truss, a type of structure sometimes used to support roofs or bridges. Trusses are composed of individual members so connected to form triangles (see Figure 1.3). The principles of statics allow one to determine the forces induced in each of the individual members.

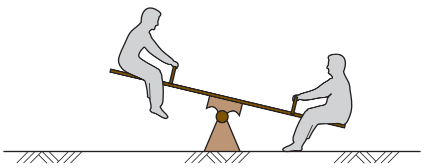


FIGURE 1.2 Teeter-totter example.

1.3 THE MATHEMATICS OF STATICS

Statics is an analytical subject that usually requires the physical conceptualization, as well as the mathematical modeling, of a problem. Complicated mathematics is not required in our treatment of the subject. A knowledge of basic arithmetic, algebra, geometry, and trigonometry is sufficient. Because of the importance of directions of forces and the geometric layout of typical structures, familiarity with trigonometry is necessary. A brief review of essential trigonometric relationships follows.

1.3.1 Right Triangles

In Figure 1.4a right triangle *ABC* is shown. The right angle (90°) at *C* is indicated. Angles *A* and *B* are acute (less than 90°) angles. As with all triangles, the sum of the three interior angles is 180°. The sides opposite angles *A*, *B*, and *C* are denoted *a*, *b*, and *c*, respectively. Side *c* is the hypotenuse of the right triangle, and the other two sides are the legs (or simply, the sides).

The ratios formed between various sides of the right triangle are termed *trigonometric functions* (or *trig functions*) of the acute angles. The functions of importance to us are the sine, cosine, and tangent. These are abbreviated as sin, cos, and tan, respectively, and are defined as follows:

$$\sin = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan = \frac{\text{opposite side}}{\text{adjacent side}}$$

From the preceding definitions, and with reference to the right triangle of Figure 1.4a, the following equations may be written:

$$\sin B = \frac{b}{c} \qquad \cos B = \frac{a}{c} \qquad \tan B = \frac{b}{a}$$

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$

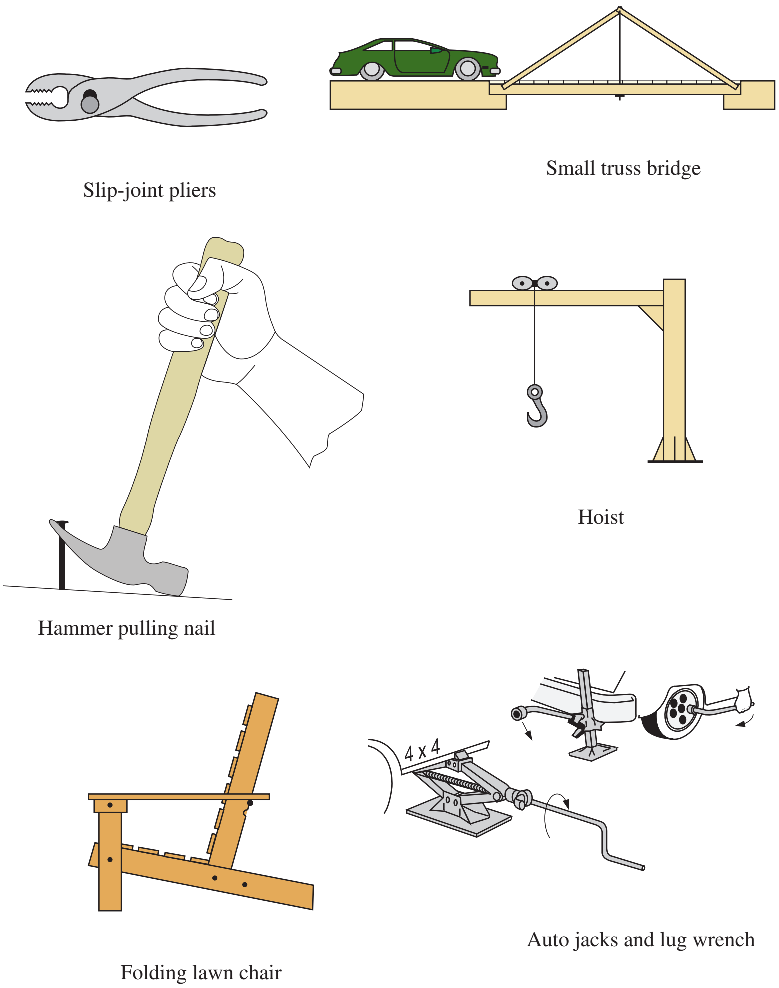
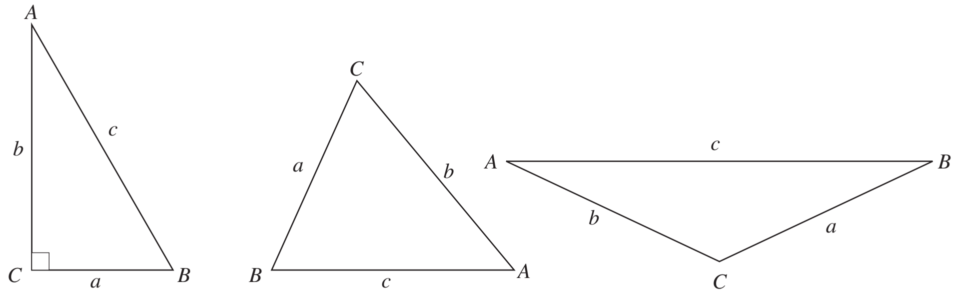


FIGURE 1.3 Everyday applications of statics.



(a) Right triangle **(b)** Oblique triangle (acute angles only) **(c)** Oblique triangle (obtuse angle and acute angles)

FIGURE 1.4 Types of triangles.

These values are constant for a given angle, regardless of the size of the triangle and can be calculated easily with a scientific calculator.

A relationship formulated by Pythagoras, a Greek philosopher and mathematician, gives us another tool for use with right triangles. The Pythagorean theorem states that in a right triangle, the square of the hypotenuse equals the sum of the squares of the other two sides. With reference to Figure 1.4a,

$$c^2 = a^2 + b^2$$

Knowing two sides of a right triangle, or one side and one of the acute angles, the unknown sides and angles can be computed using the Pythagorean theorem and/or the trig functions.

1.3.2 Oblique Triangles

An *oblique triangle* is one in which no interior angle is equal to 90°. It may have three acute (less than 90°) angles, or two acute angles and one obtuse (greater than 90°) angle, as shown in Figure 1.4b and c. As with the right triangle, the sum of the three interior angles is 180°.

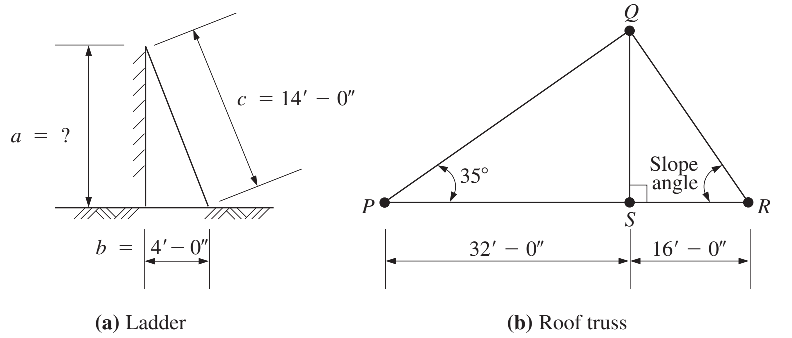


FIGURE 1.5 Mathematics of statics examples.

Knowing three sides, or two sides and the included angle, or two angles and the included side, the unknown sides and angles can be computed using the following laws:

1. The law of cosines:

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$b^2 = a^2 + c^2 - 2ac(\cos B)$$
$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

2. The law of sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The letter designations are shown in Figure 1.4b and c.

The following examples illustrate solutions of both the right triangle and oblique triangle. (Refer to Figure 1.5 for Examples 1.1 and 1.2.)

EXAMPLE 1.1 A 14-ft-long ladder leans against a wall with the bottom of the ladder placed 4 ft from the base of the wall, as shown in Figure 1.5a. How high on the wall will the ladder reach?

Solution The Pythagorean theorem is used:

$$c^2 = a^2 + b^2$$

Rewrite, substitute, and solve for a:

$$a^2 = c^2 - b^2 = (14\text{ ft})^2 - (4\text{ ft})^2 = 180\text{ ft}^2$$

from which

$$a = \sqrt{180\text{ ft}^2} = 13.42\text{ ft}$$

Conversion of this result from decimal feet to feet and fractional inch units is discussed in Section 1.5.

EXAMPLE 1.2 For the roof truss shown in Figure 1.5b, determine the height QS, the length of the steep slope QR, and the slope angle at R.

Solution To determine QS, use the triangle PQS:

$$\tan 35° = \frac{\text{opposite}}{\text{adjacent}} = \frac{QS}{32}$$
$$QS = 32\text{ ft}(\tan 35°) = 22.4\text{ ft}$$

To determine QR, use the Pythagorean theorem for triangle QRS:

$$(QR)^2 = (QS)^2 + (SR)^2$$
$$= (22.4\text{ ft})^2 + (16\text{ ft})^2 = 758\text{ ft}^2$$
$$QR = 27.5\text{ ft}$$

Now find the angle at R using any of the three trig functions (since all three sides of the triangle QRS are known):

$$\tan R = \frac{\text{opposite}}{\text{adjacent}} = \frac{22.4\text{ ft}}{16\text{ ft}} = 1.40$$

Determine the angle that has a tangent of 1.40. This is called the *arc tangent* of 1.40 and is written as

$$R = \tan^{-1}(1.40)$$
$$R = 54.5°$$

or

$$R = \sin^{-1}\left(\frac{\text{opposite}}{\text{hypotenuse}}\right) = \sin^{-1}\left(\frac{22.4\text{ ft}}{27.5\text{ ft}}\right) = 54.5°$$

or

$$R = \cos^{-1}\left(\frac{\text{adjacent}}{\text{hypotenuse}}\right) = \cos^{-1}\left(\frac{16\text{ ft}}{27.5\text{ ft}}\right) = 54.4°$$

The slight difference in the solution is due to rounding and can be neglected.

EXAMPLE 1.3 Compute the angle B between cables AB and BC if a force (load) is applied as shown in Figure 1.6.

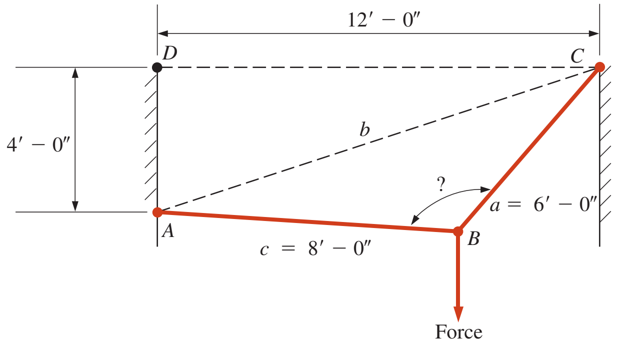


FIGURE 1.6 Cable structure.

Solution The sides of triangle ABC (which is *not* a right triangle) have been designated a, b, and c, as shown. Compute distance AC using right triangle ACD and the Pythagorean theorem:

$$(AC)^2 = (12\text{ ft})^2 + (4\text{ ft})^2$$
$$AC = 12.65\text{ ft}$$

Compute angle B using oblique triangle ABC and the law of cosines:

$$b^2 = a^2 + c^2 - 2ac(\cos B)$$
$$12.65^2 = 6^2 + 8^2 - 2(6)(8)\cos B$$
$$\cos B = -0.625$$

Therefore,

$$B = \cos^{-1}(-0.625) = 128.7°$$

If you recall the rules of trigonometry, the cosine of an angle can only be negative if the angle is between 90° and 270°. Through visual inspection, it is clear that the angle is less than 180°, so the answer of 128.7° makes sense. Asking yourself if the answer is reasonable should be the done after every calculation.

EXAMPLE 1.4 A rigging boom is supported by means of a boom cable *BC* as shown in Figure 1.7. Compute the length of the cable and the angle it makes with the boom (angle *C* in oblique triangle *ABC*).

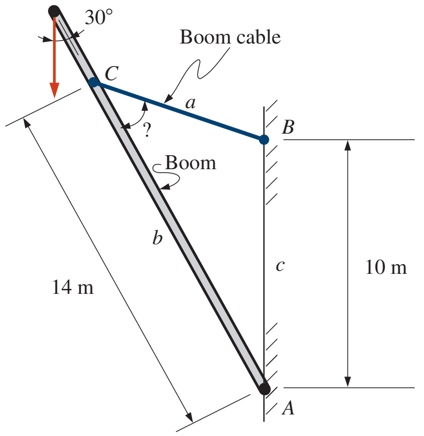


FIGURE 1.7 Cable-supported boom.

Solution The sides of triangle *ABC* are designated *a, b*, and *c*, as shown. Compute the length of the boom cable *a* using the law of cosines. Note that the data needed for the law of cosines are two sides and the included angle and that the side to be found is opposite the known angle. Also note that angle $A = 30°$ by alternate interior angles.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc(\cos A) \\ &= (14\text{ m})^2 + (10\text{ m})^2 - 2(14\text{ m})(10\text{ m})\cos 30° \\ &= 53.51\text{ m}^2 \end{aligned}$$

from which

$$a = 7.32\text{ m}$$

Then compute the angle that the cable makes with the boom (angle *C*) using the law of sines:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{7.32\text{ m}}{\sin 30°} = \frac{10.0\text{ m}}{\sin C}$$

$$\sin C = \frac{10.0\text{ m}(\sin 30°)}{7.32\text{ m}} = 0.683$$

from which

$$C = \sin^{-1}(0.683) = 43.1°$$

1.3.3 Solving Simultaneous Equations

In addition to a required familiarity with trigonometry, one must also be familiar with various algebraic manipulations and equations. One type of problem that is often encountered involves the need to solve for two or more unknown quantities that are related by linear equations. Such equations are called *simultaneous equations*.

The following examples will illustrate two solution methods for this type of problem.

EXAMPLE 1.5 An engineer lives 5 mi from his office. There is a new bicycle ride sharing initiative in his city. In an attempt to get some regular exercise, he decides to jog to a bicycle sharing location each morning and then ride a bicycle for the remainder of the distance. He knows that he can average 18 mph on the bike and 6.0 mph jogging.

He would like to get to work in one-half hour. How long should he ride and how long should he jog?

Solution Let

$$x = \text{the length of time to ride (hr)}$$
$$y = \text{the length of time to jog (hr)}$$

The two equations may be expressed as follows:

$$x + y = 0.5 \text{ hr} \quad \textbf{(1.1)}$$

and

$$18 \text{ mph } (x) + 6 \text{ mph } (y) = 5.0 \text{ mi}$$

from which

$$18x + 6y = 5.0 \text{ mi} \quad \textbf{(1.2)}$$

Algebraic solution method: To eliminate one unknown, multiply Equation 1.1 by −6 and then add the two equations:

$$\begin{aligned} -6x - 6y &= -3.0 \\ +18x + 6y &= +5.0 \\ \hline +12x \quad &= +2.0 \end{aligned}$$

from which

$$x = 0.1667 \text{ hr}$$

Since $x + y = 0.5$, substitute for x:

$$\begin{aligned} 0.1667 + y &= 0.5 \\ y &= 0.5 - 0.1667 \\ y &= 0.333 \text{ hr} \end{aligned}$$

Substitution method: Solve one of the equations for one of the variables and substitute this expression into the other equation.

Solve Equation 1.1 for y:

$$\begin{aligned} x + y &= 0.5 \text{ hr} \\ y &= 0.5 - x \end{aligned}$$

Substitute this expression into Equation 1.2:

$$\begin{aligned} 18x + 6y &= 5.0 \text{ mi} \\ 18x + 6(0.5 - x) &= 5.0 \\ 18x + 3.0 - 6x &= 5.0 \\ +12x &= 2.0 \\ x &= 0.1667 \text{ hr} \end{aligned}$$

Then

$$\begin{aligned} y &= 0.5 - x \\ y &= 0.5 - 0.1667 = 0.333 \text{ hr} \end{aligned}$$

1.4 CALCULATIONS AND NUMERICAL ACCURACY

Solutions to problems cannot be more accurate than the engineering data that are used. When dealing with statics problems, we must consider that the dimensions of structural and machine parts and the loads used in the analysis are accurate only to a certain degree. While calculation methods and tools are capable of handling numbers having many digits, this degree of accuracy is usually not warranted.

A *significant digit* is a meaningful digit, one that reflects a quantity that has been measured (or counted) and is thought to be accurate. The accuracy of a number is implied by the significant digits shown. The number 58 has two significant digits, while the number 7 has only one significant digit.

An ordinary calculator will yield the following result:

$$\frac{58}{7} = 8.285714286$$

The 10 digits of the result shown, however, indicate an accuracy far greater than that of the numbers going into the calculation (one significant digit in the denominator). Logically the result of a calculation should not reflect an accuracy greater than that of the data from which that result is obtained. Therefore, one should guard against implying unwarranted accuracy in this manner and simply round the result of the preceding calculation to 8 (not 8.0 or 8.00, since these contain two and three significant digits, respectively).

To determine the number of significant digits in a number, begin at the left with the first nonzero digit and count left to right across the number. Stop counting at the last nonzero digit unless any trailing zeros are to the right of the decimal point, in which case they are considered significant. Note that the location of the decimal point does not establish the number of significant digits. For example, each of the following numbers has three significant digits:

$$4.78 \qquad 47.8 \qquad 0.478 \qquad 0.00478 \qquad 4.78 \times 10^6 \qquad 0.470$$

A predicament may exist with a number such as 47,800. The usual assumption is that the two zeros exist to place the decimal point only and are not significant. Yet, the possibility exists that the two zeros were measured or counted and that there are actually five significant digits. One way around this problem is to use exponential notation:

$$478 \times 10^2 \text{ (three significant digits)}$$
$$478.00 \times 10^2 \text{ (five significant digits)}$$

Although it may be difficult to attest to the accuracy of the data available, it is generally agreed that engineering data are rarely known to an accuracy of greater than 0.2%, or 1 in 500. When dealing with structural calculations, this is equivalent to a possible error of 100 lb in a 50,000-lb load:

$$\frac{100}{50{,}000}(100) = 0.2\%$$

Rather than tediously determining 0.2% of each numerical solution, a general rule of thumb for engineering calculations has evolved: *Represent solutions numerically to an accuracy of three significant digits. If the number begins with 1, then use four significant digits*. This rule keeps us true to the spirit (if not within the letter) of the 0.2% guideline.

In rounding numbers, the following method is used:

1. If the digit to be dropped is 5 or greater, the digit to the left is increased by 1. For example, 47.68 becomes 47.7.
2. If the digit to be dropped is less than 5, the digit to the left remains unchanged. For example, 47.62 becomes 47.6.

Adhering to the preceding method, we would round as follows:

18,435.35 rounds to 18,440
2.2321 rounds to 2.23
0.096831 rounds to 0.0968

We have attempted to maintain consistency in our problem solutions by rounding intermediate and final numerical solutions in accordance with this method. For the text presentation, we have used the rounded intermediate solution in the subsequent calculations. Ideally, when working on a calculator or computer, however, one would normally maintain all digits and round only the final answer. For this reason, the reader may frequently obtain numerical results that differ slightly from those given in the text. These differences should not cause concern.

1.5 CALCULATIONS AND DIMENSIONAL ANALYSIS

An integral part of the calculation process in mechanics deals with the proper handling of units. In most cases (not all), a unit must be included with a numerical result to accurately describe the quantity in question. If the result is to be a calculated distance, for example, the associated unit must be a length unit (ft, in., m, etc.).

Thoroughness in the calculation process, particularly for the beginner in the mechanics field, should incorporate inclusion of all units in the calculation. For example, the simple conversion of 87.3 ft to miles (mi) can be calculated as follows:

$$87.3\ \not{ft}\left(\frac{1\text{ mi}}{5280\ \not{ft}}\right) = 0.01653\text{ mi}$$

Notice that the foot units of the original quantity will be cancelled by the foot units in the denominator of the conversion factor (within the parentheses) since ft/ft = 1.

Therefore, the resulting unit will be the mile. The cancelling process is indicated by strike marks through the units.

In a similar example, when converting 185 yards to miles, using familiar conversions,

$$185\ \not{yd}\left(\frac{3\ \not{ft}}{1\ \not{yd}}\right)\left(\frac{1\text{ mi}}{5280\ \not{ft}}\right) = 0.1051\text{ mi}$$

The inclusion of units within calculations is always recommended. When all units are included, they point out any necessary conversions and occasionally also provide clues as to substitution errors. Some conversions and equivalencies that will be useful include are as follows:

1 mile (mi)	=	5280 ft
1 mi^2	=	640 acres
1 acre	=	43,560 ft^2
1 yard (yd)	=	3 ft
1 rod	=	5.50 yd
1 kip (k)	=	1000 lb
1 ton (short ton)	=	2000 lb
1 ft^3 fresh water	=	62.4 lb
1 ft^3	=	7.481 gallons (gal)

EXAMPLE 1.6 Determine the weight (in kips) of water contained in a cylindrical tank having a radius r of 20 in. and a height h of 5 ft. (*Note:* The kip [short for kilopound] is a unit of force and equals 1000 lb.)

Solution The weight of the water is calculated from

$$\text{weight} = \text{unit weight} \times \text{volume}$$

Note that the unit weight of freshwater is 62.4 lb/ft^3. Therefore, the volume must be found in units of cubic feet (ft^3), and any length units of inches must be converted to units of feet. The volume of the cylinder is determined from

$$V = \pi r^2 h$$

Therefore,

$$\begin{aligned}\text{weight} &= \left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(\pi)(20\text{ in.})^2\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2(5\text{ ft})\left(\frac{1\text{ kip}}{1000\text{ lb}}\right)\\ &= 2.72\text{ kips}\end{aligned}$$

Note in the final equation of Example 1.6 that the fourth term on the right side converts the square of the radius (in.^{2}) to square feet (ft^2).

When using the U.S. Customary (inch-pound) System of units for construction applications, it is common to express dimensions in feet and fractional inches. Calculations, however, are carried out using decimal feet (or decimal inches).

EXAMPLE 1.7 In Example 1.1, the solution for the vertical leg of the right triangle was 13.42 ft. This was a rounded value. Convert this decimal feet dimension to feet and fractional inches.

Solution

1. The 13.42 ft dimension contains 13 full feet and this will not change.
2. The remaining 0.42 ft must be converted to decimal inches.

$$0.42\text{ ft} \times 12\text{ in./ft} = 5.04\text{ in.}$$

3. Thus far, then, we have 13 ft and 5 (full) inches, and we need now to convert the remaining 0.04 in. to fractional inches. The accuracy of the fraction (the number used for the denominator of the fraction) depends on the application. For instance, in structural steel fabrication, dimensions to $\frac{1}{16}$ in. are common. To convert 0.04 in. to sixteenths of an inch, multiply by 16 (there are 16 sixteenths of an inch per inch):

$$0.04\text{ in.} \times 16 = 0.64\text{ (sixteenths of an inch)}$$

and round 0.64 (sixteenths) to 1 (sixteenth).

4. Therefore, summing the results of steps 1, 2, and 3:

$$13.42\text{ ft} = 13\text{ ft} + 5\text{ in.} + \tfrac{1}{16}\text{ in.} = 13\text{ ft} - 5\tfrac{1}{16}\text{ in. or } 13' - 5\tfrac{1}{16}''$$

(*Note*: The dash is common usage and should not be interpreted as a minus sign.)
If we want to show 13.42 ft to an accuracy of $\frac{1}{64}$ in., step 3 would change to:

$$0.04 \times 64 = 2.56\text{ (sixty-fourths)}$$

Therefore,

$$13.42\text{ ft} = 13\text{ ft} - 5\,\tfrac{3}{64}\text{ in.}$$